Large scale magnetic fields and Dynamo theory

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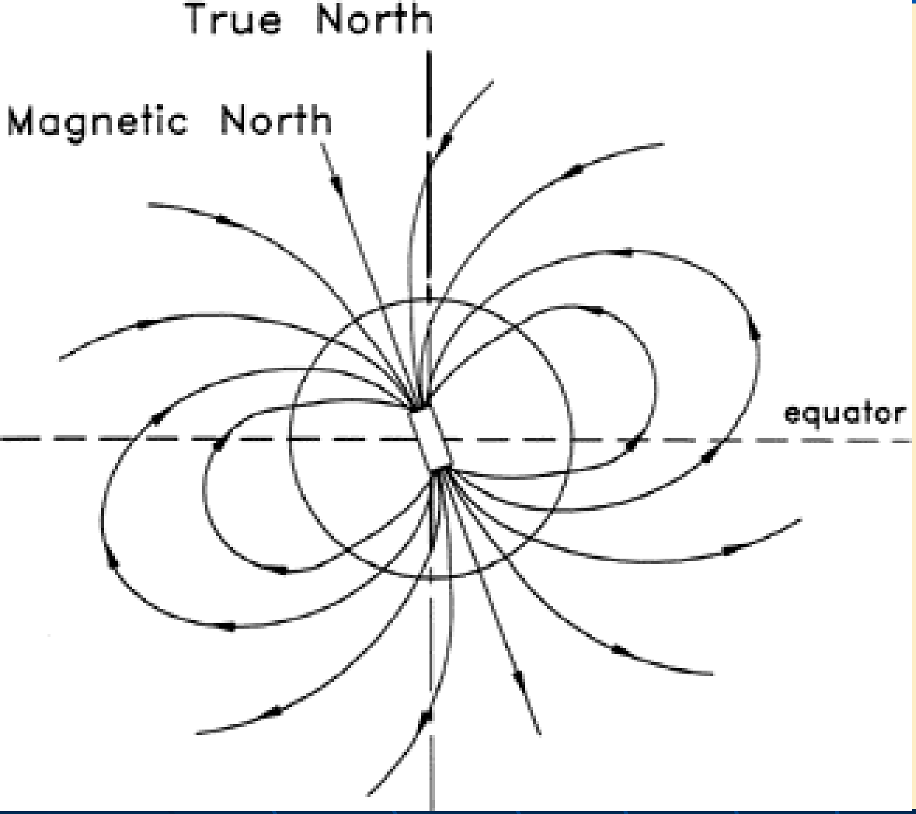
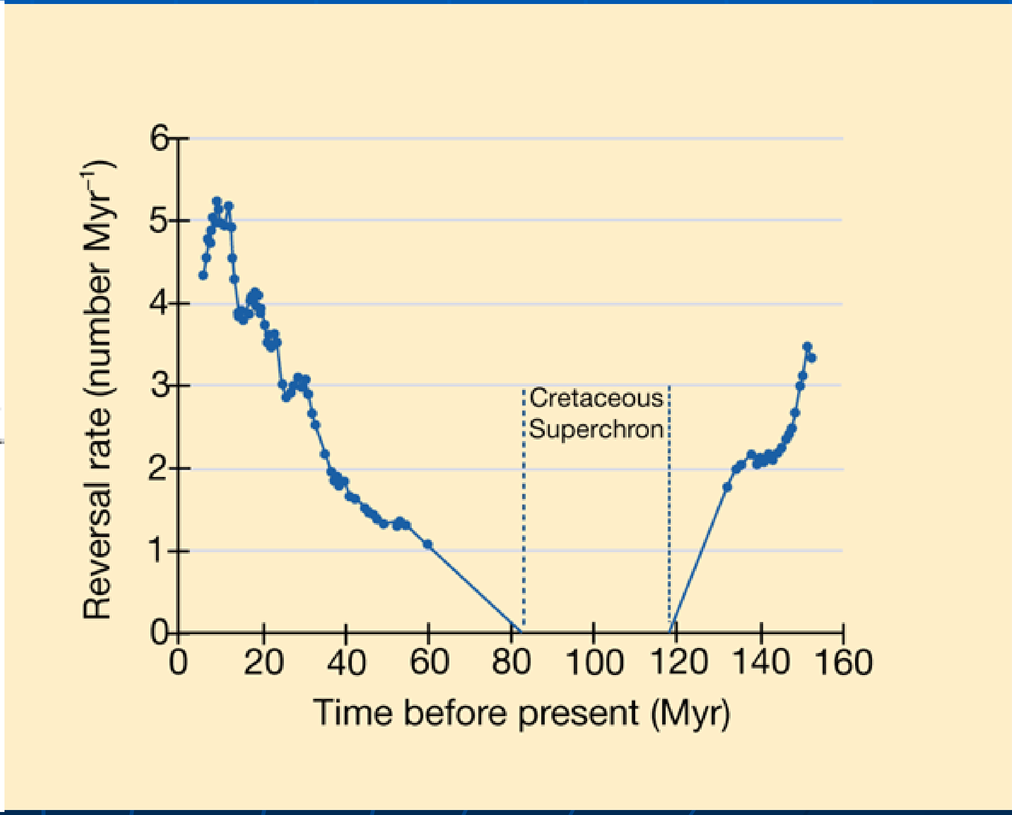
The Earth

- Mainly dipolar magnetic field
- Would decay in 20kyr if not regenerated
- Declination of the dipole axis – 11.5° (at present)
- Strength – B=0.3-0.6G (at present)
- Field reversal – interchange of North/South poles, last ~300-3000yr
- Field excursion – decrease of B to 0-20% of normal, last ~3000yr-0.03Myr

There are successful simulations: Glatzmaier, Roberts 1996+

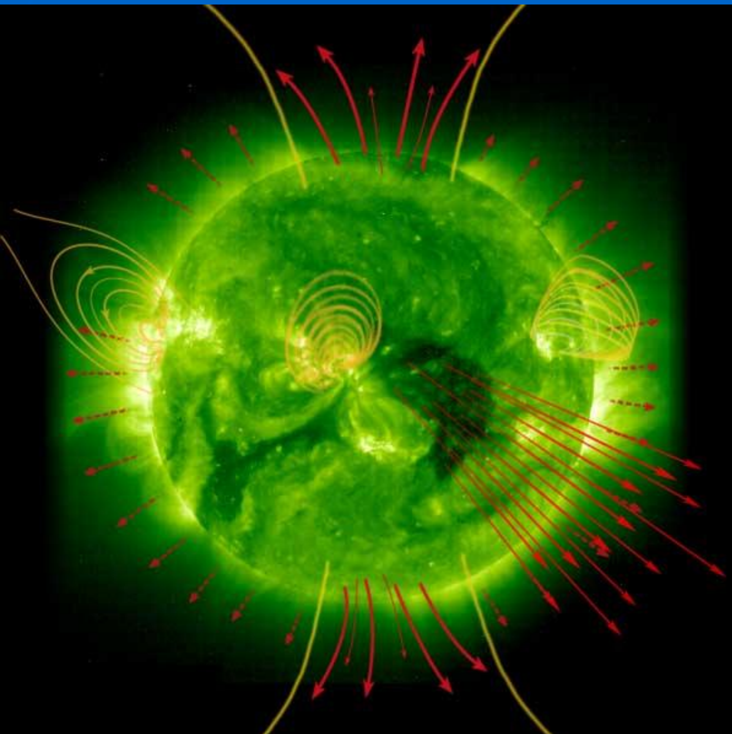




http://solar.physics.montana.edu/ypop/Spotlight/Magnetic/what.html

Buffett, 1999 Nature 401, 861

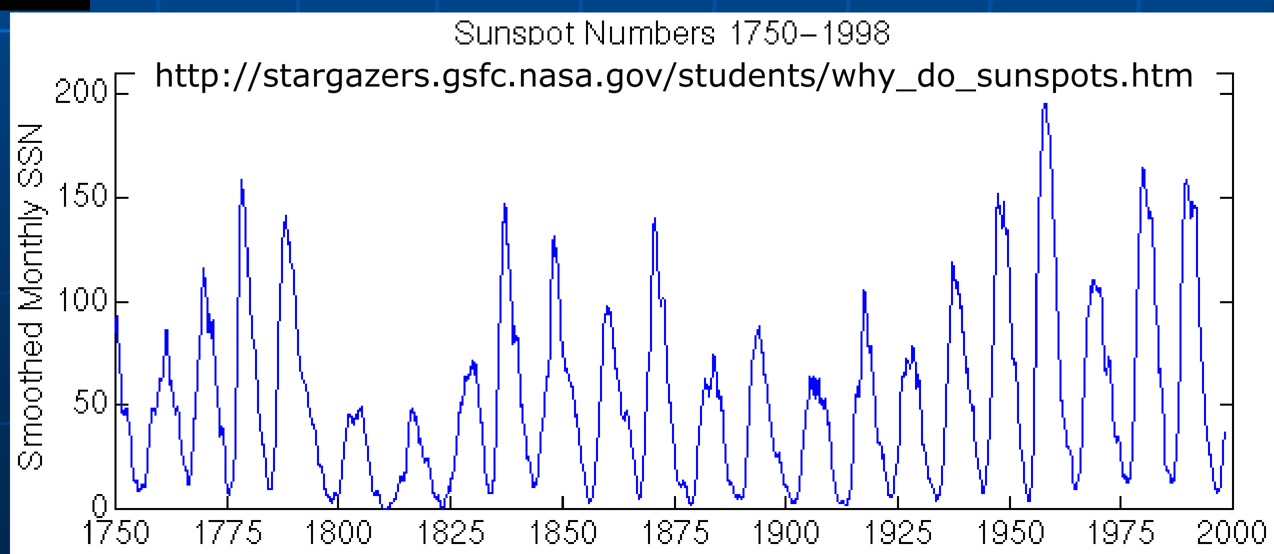
The Sun



- Strong non-dipolar magnetic field
- Typical field strength – B~1G, up to 1000G in active regions
- Field reversal – 22-year cycle, sunspots – 11-year cycle (7-15 years for period)
- Has irregular outflows driven partially by magnetic reconnections
- Differential rotation – 25 days (equator) vs 35 days (pole)

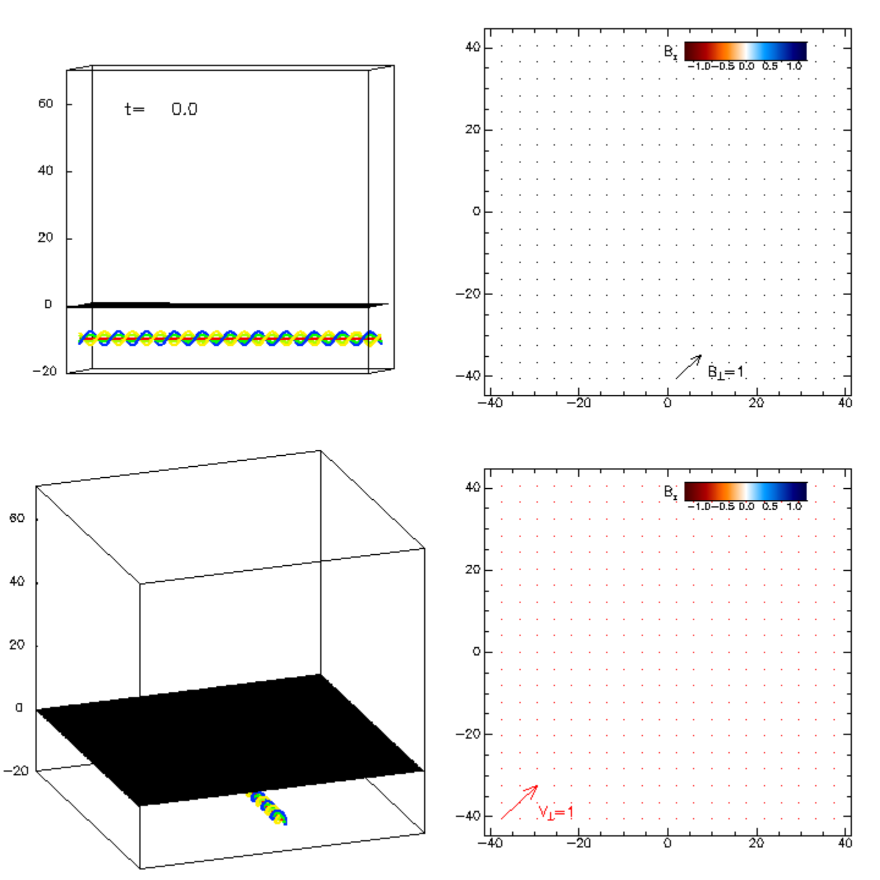
http://www.spacedaily.com/reports/The_Magnetic_Fields_of_Planets_and_Stars_999.html

Models only with fitted parameters explain observations

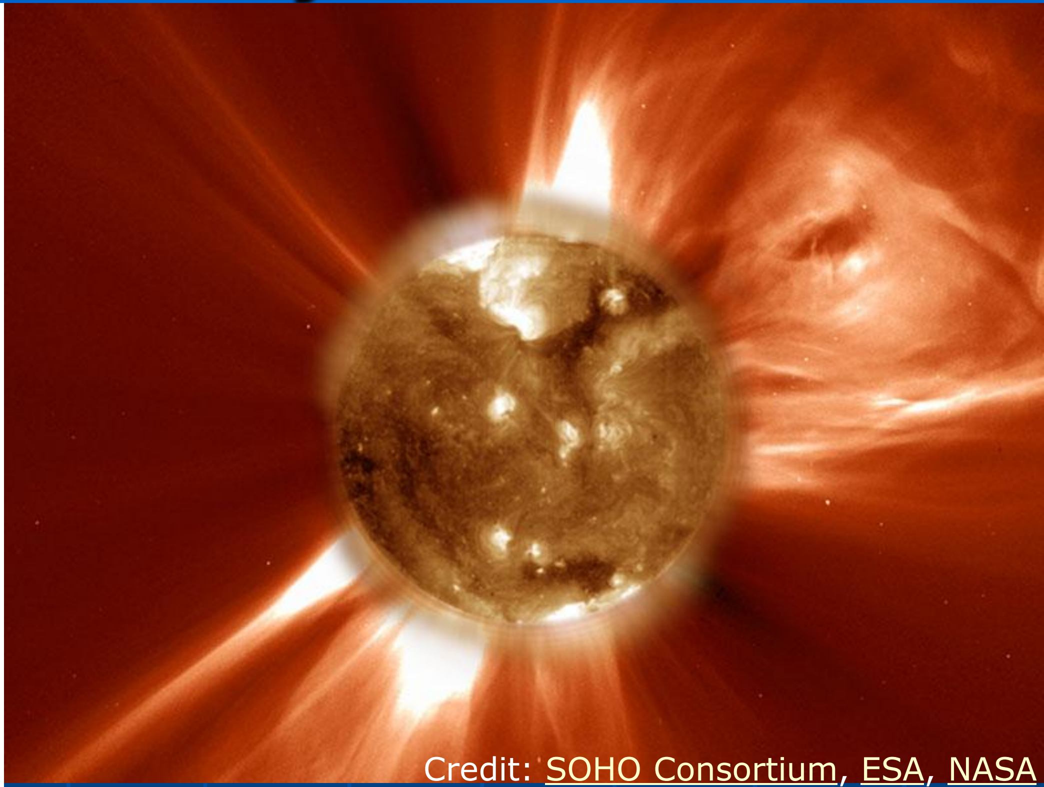


http://stargazers.gsfc.nasa.gov/students/why_do_sunspots.htm

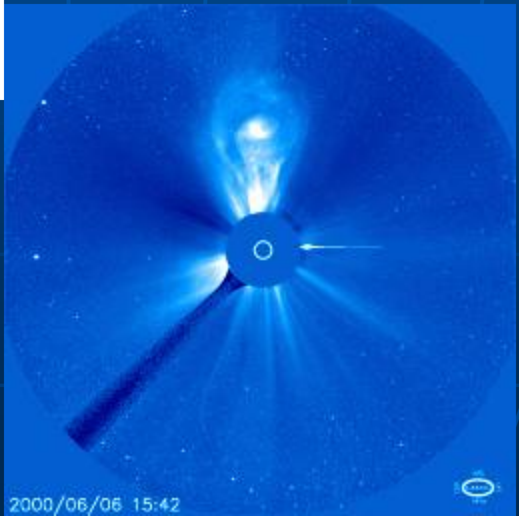
Coronal mass ejections



Fan 2001, ApJ, 554, L111



Credit: SOHO Consortium, ESA, NASA



http://www.space.com/businesstechnology/technology/technovel_shock_041105.html

May be the constituent part of dynamo

Energy sources

The Earth and planets

- Radioactive decay – $^{40}K,\ ^{238}U$
- Latent heat & light constituents release – inner core freezing
- Precessionally driven flows
- Tidal heating

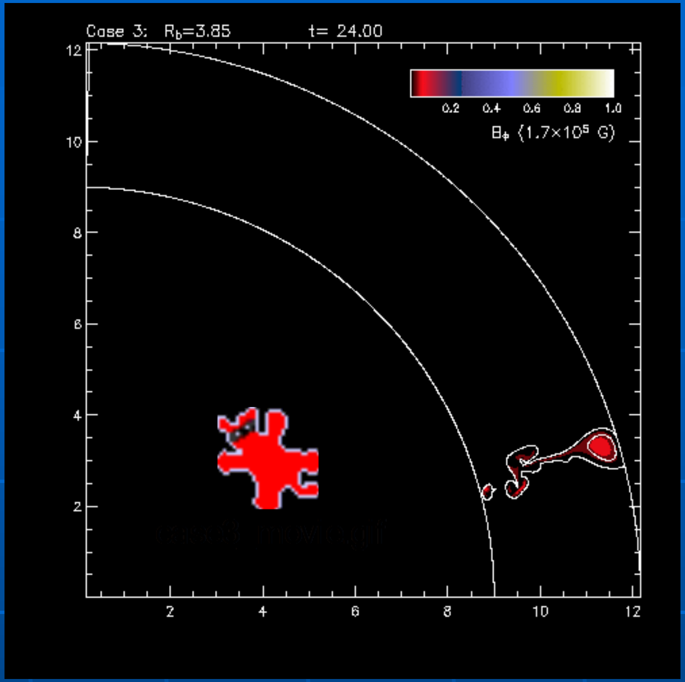
→ **Convection**

→ **Induced flows**

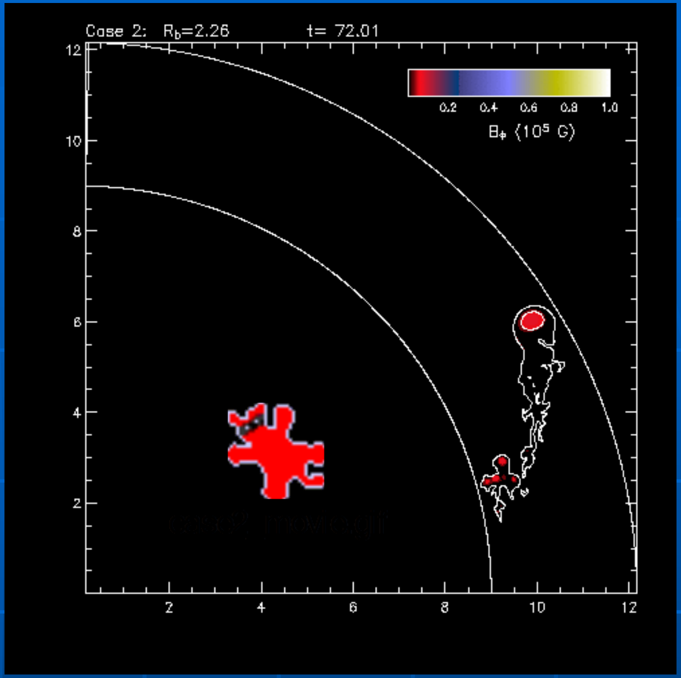
The Sun and stars

- Nuclear burning
- Dynamical interactions in binaries
- Primordial magnetic field

Convection in rotating medium



$R_b = v_{A0} / 2\Omega H_p$



Rotation decreases convective efficiency

Back-reaction of convection accelerates rotation

Differential rotation

Dynamo action

Timescales and their ratios

Reynolds number

$$\mathbf{Re} = \frac{UL}{\nu} = \frac{viscous\ \ time}{dynamical\ \ time} = \frac{inertial\ \ force}{viscous\ \ force} >> 10^3$$

Onset of turbulence

Prandtl number (magnetic)

$$\mathbf{Pm} = \frac{\nu}{\eta} = \frac{Ohmic\ time}{viscous\ time} = \frac{viscous\ dissipatio\ n}{current\ dissipatio\ n} << 1$$

Dominance of current dissipation

Ekman number

$$E = \frac{\nu}{\Omega L^2} = \frac{rotation\ time}{viscous\ time} = \frac{viscous\ \ force}{Coriolis\ \ force} << 10^{-3}$$

Size of the boundary layer

Rayleigh number

$$Ra = \frac{(viscous\ \ time) \bullet (conduction\ \ time)}{(convection\ \ time)^2} >> 10^3$$

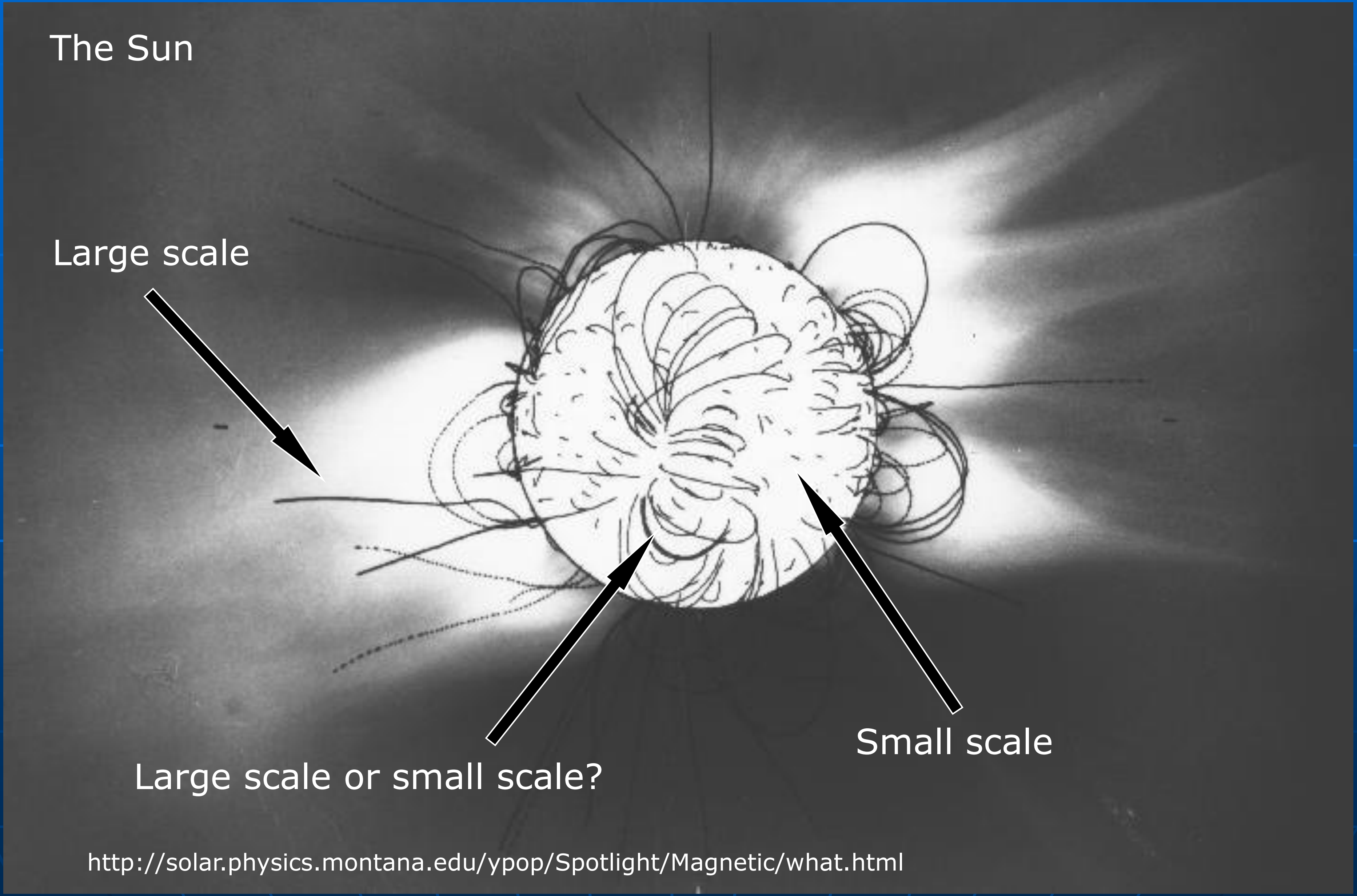
Onset of convection

Rossby number

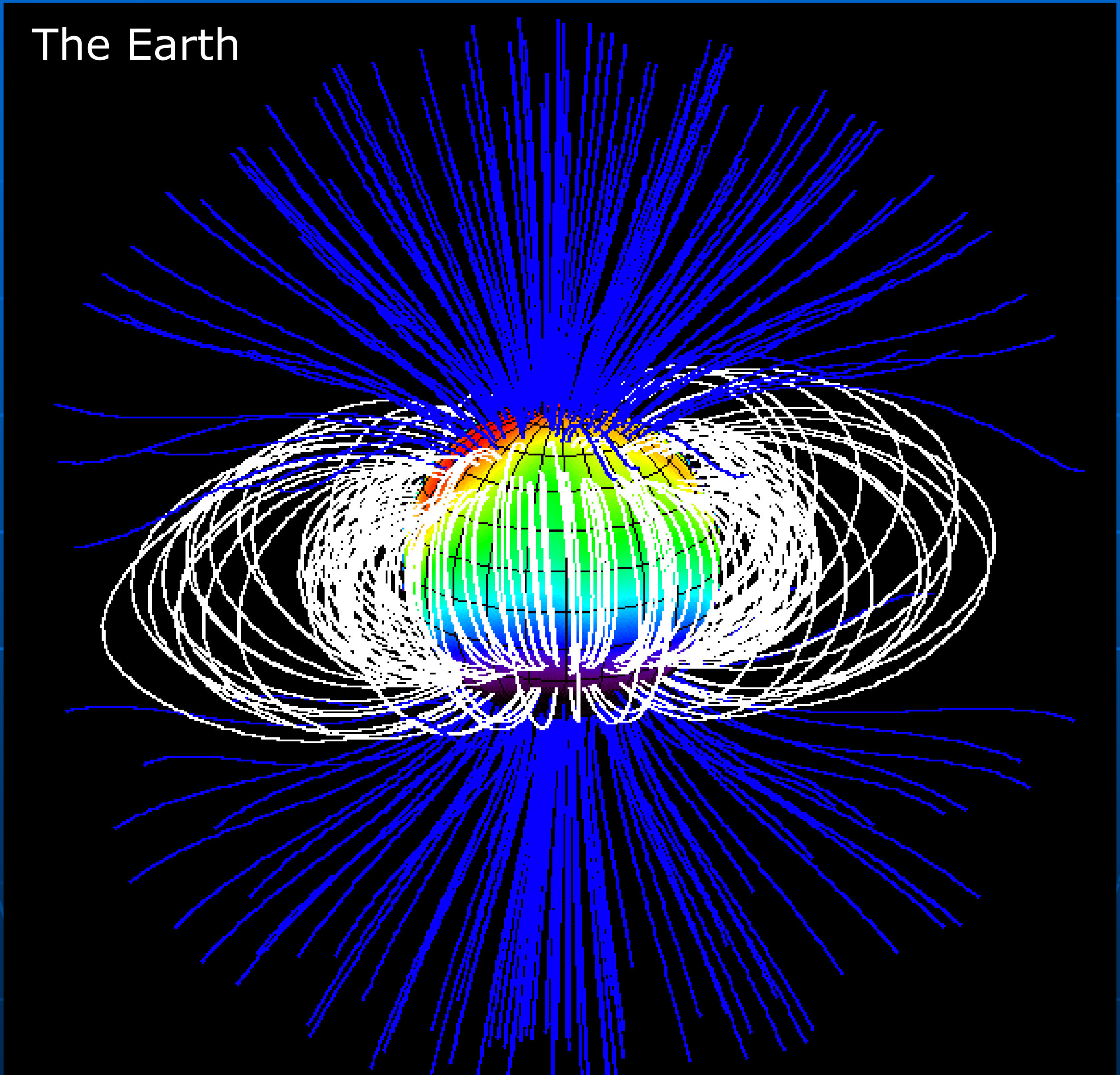
$$Ro = \frac{U}{\Omega L} = \frac{rotation\ time}{dynamical\ time} = \frac{inertial\ \ force}{Coriolis\ \ force} < 1$$

Regime and efficiency of dynamo

Large scale of small scale?



Axisymmetric or 3D?



Credit & Copyright: Gary A. Glatzmaier (UCSC)

Theory of Dynamo

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{U} \times \vec{B} + \vec{E}_M \right)$$

$$\vec{E}_M = \alpha \vec{B} - \beta \vec{J} + \gamma \vec{\Omega} + nonlinear$$

Usually neglected

Force-free regime
+diffusion

Twisting and folding

(Differential) rotation

$$\alpha \propto \left(-\vec{u} \cdot \vec{\nabla} \times \vec{u} + \vec{b} \cdot \vec{\nabla} \times \vec{b} \right)$$

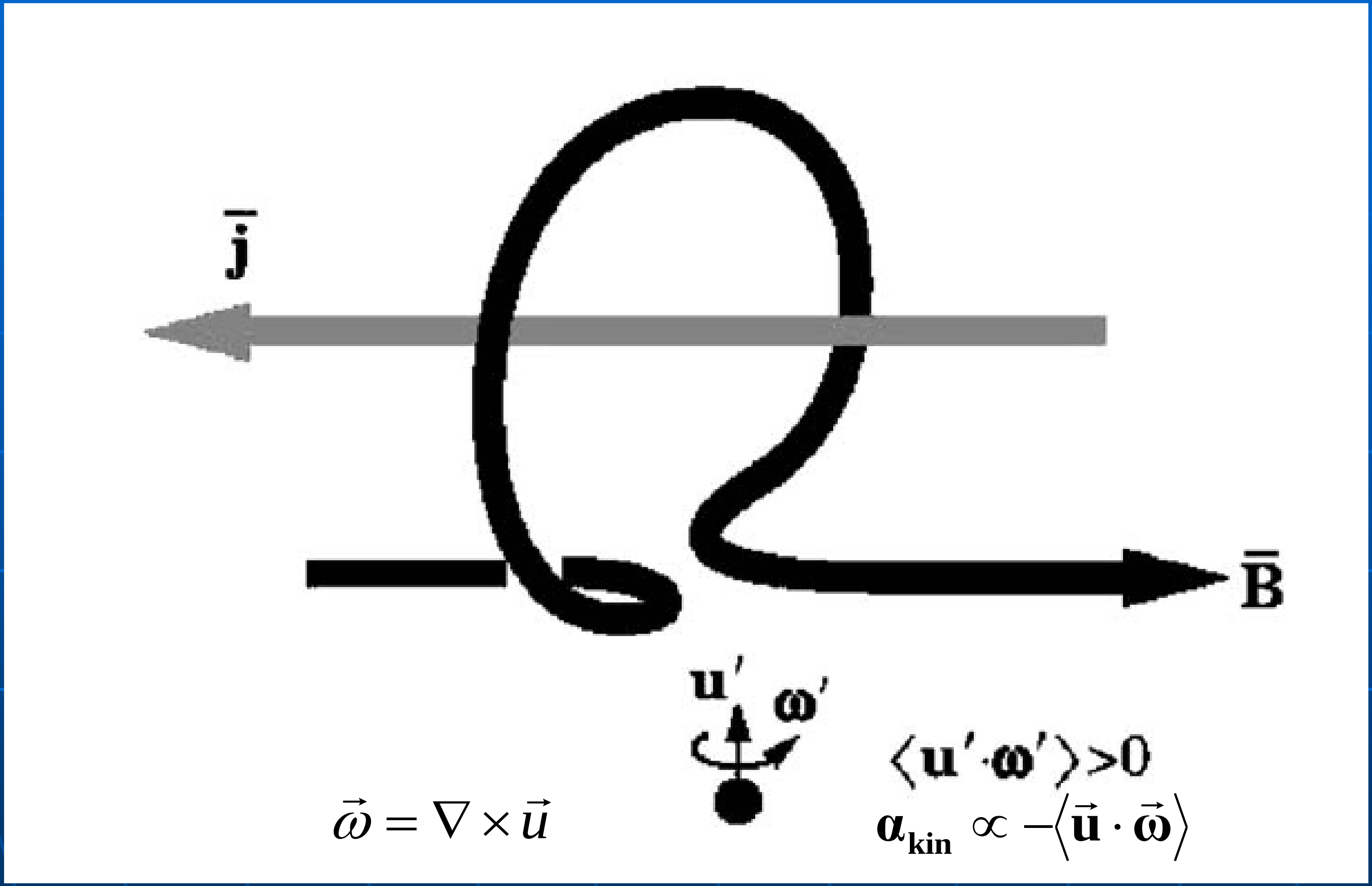
$$\gamma \propto \left(\vec{u} \cdot \vec{b} \right)$$

kinetic helicity

current helicity

cross-helicity

Kinematic α-effect



Then two possibilities:
repeat along horizontal axis – α^2 dynamo
shear along horizontal axis – $\alpha\Omega$ dynamo

Kinematic α²-dynamo

A. Brandenburg, K. Subramanian / Physics Reports 417 (2005) 1–209

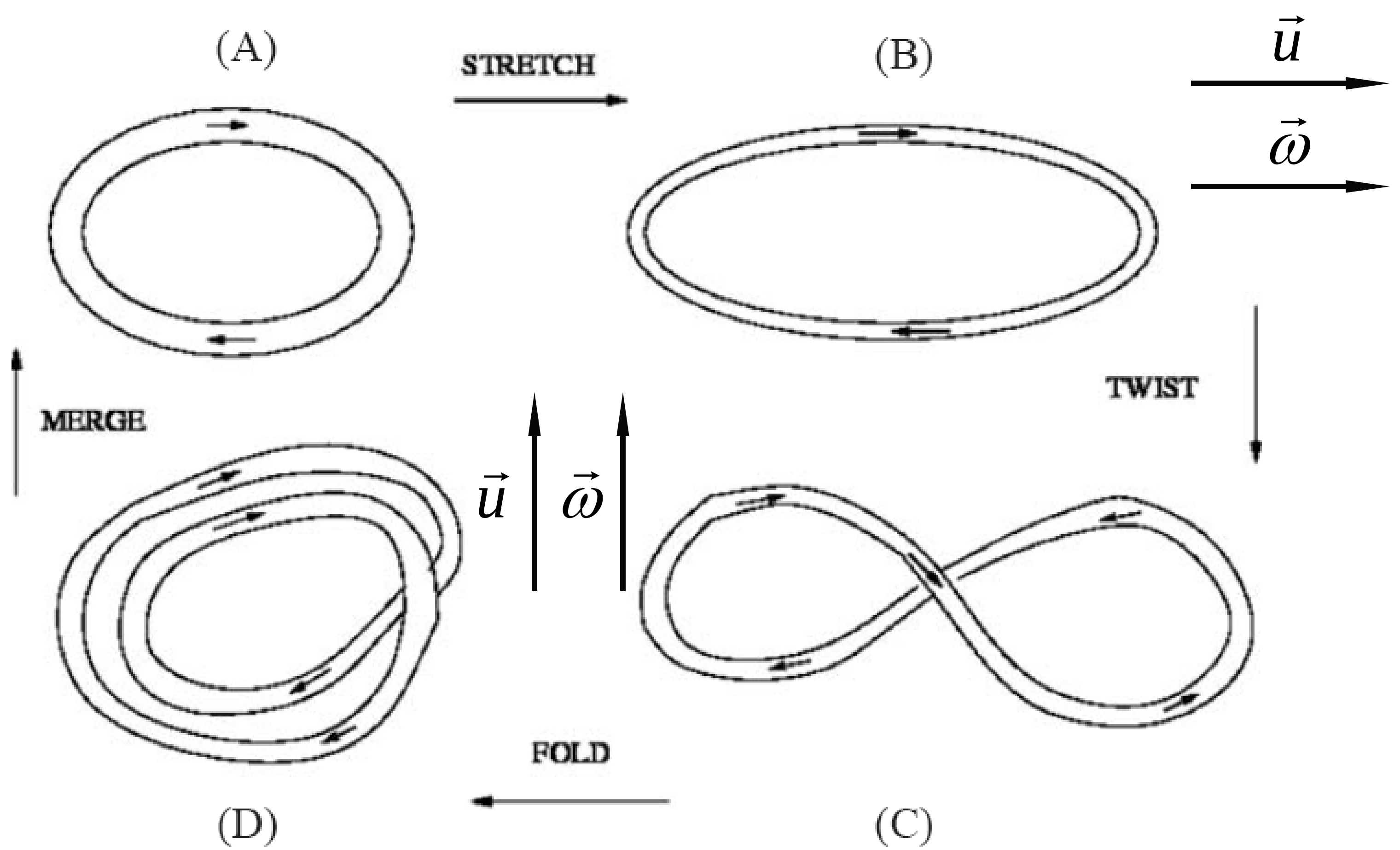


Fig. 4.6. A schematic illustration of the stretch-twist-fold-merge dynamo.

Kinematic and dynamic αΩ dynamo

Brandenburg, Subramanian, 2005

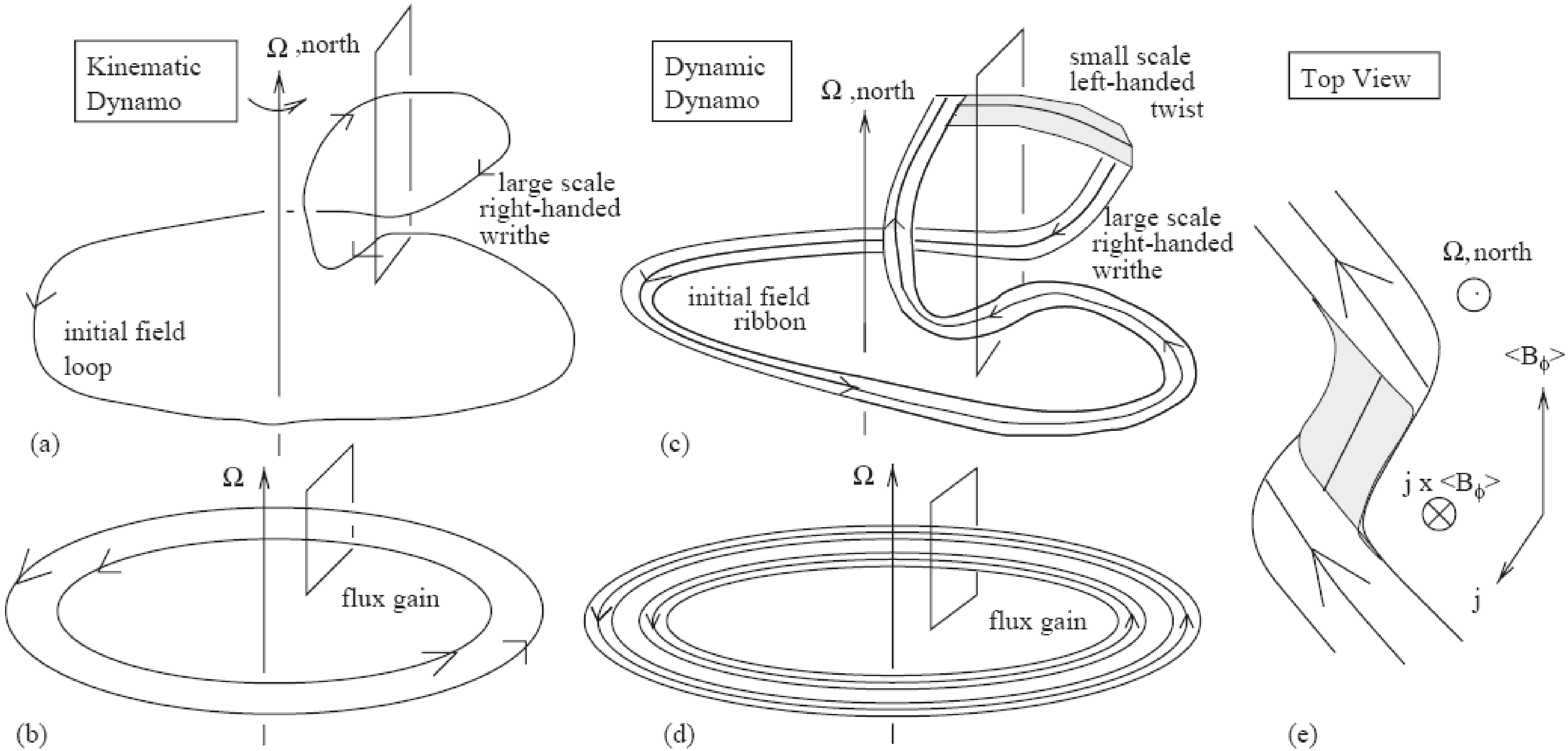
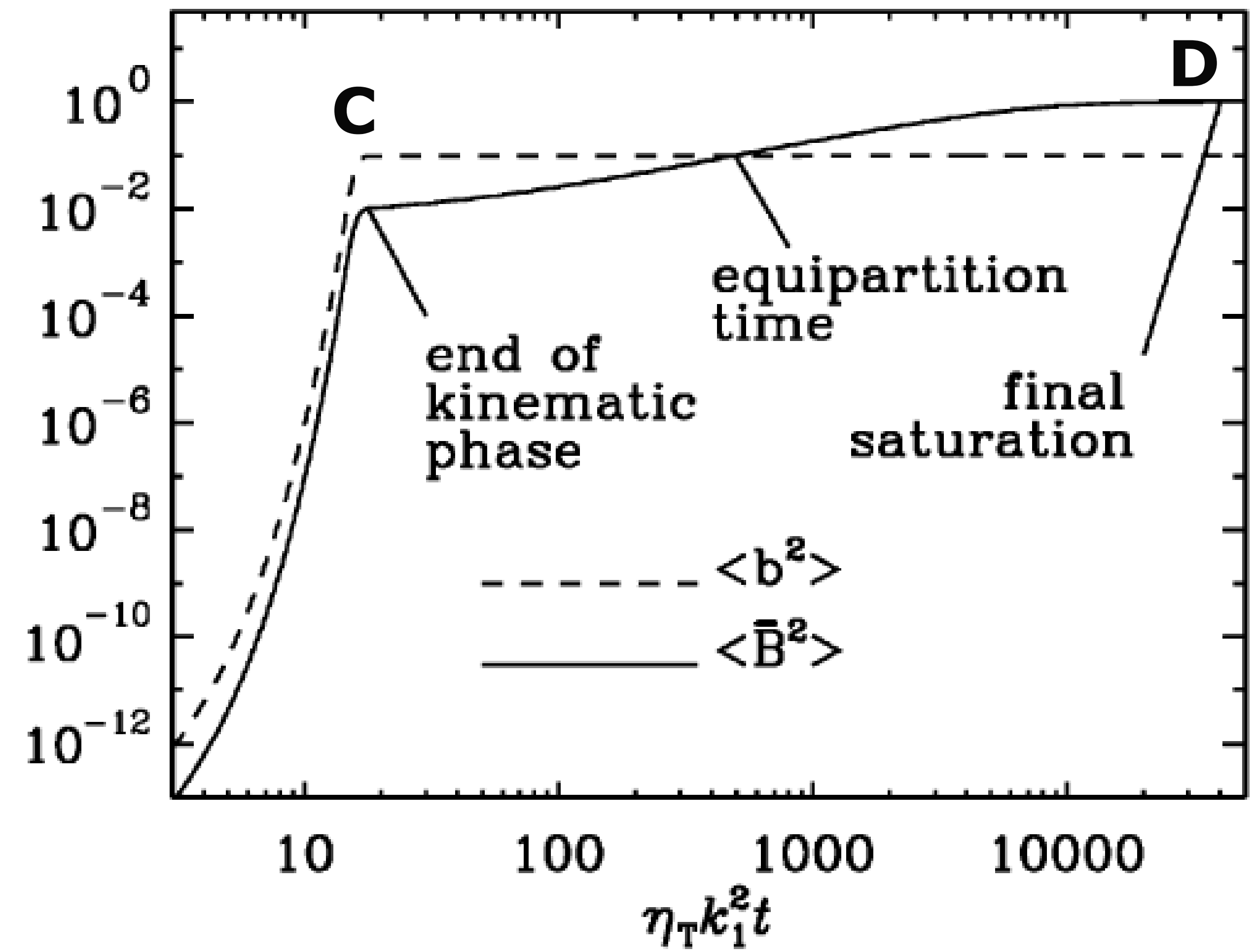


Fig. 9.6. Schematic of *kinematic* helical αΩ dynamo in northern hemisphere is shown in (a) and (b), whilst the *dynamic* helical αΩ

Strong back-reaction of the magnetic field inhibits dynamo $\alpha \to 0$

Saturation of dynamos

Brandenburg, Subramanian, 2005, p.122

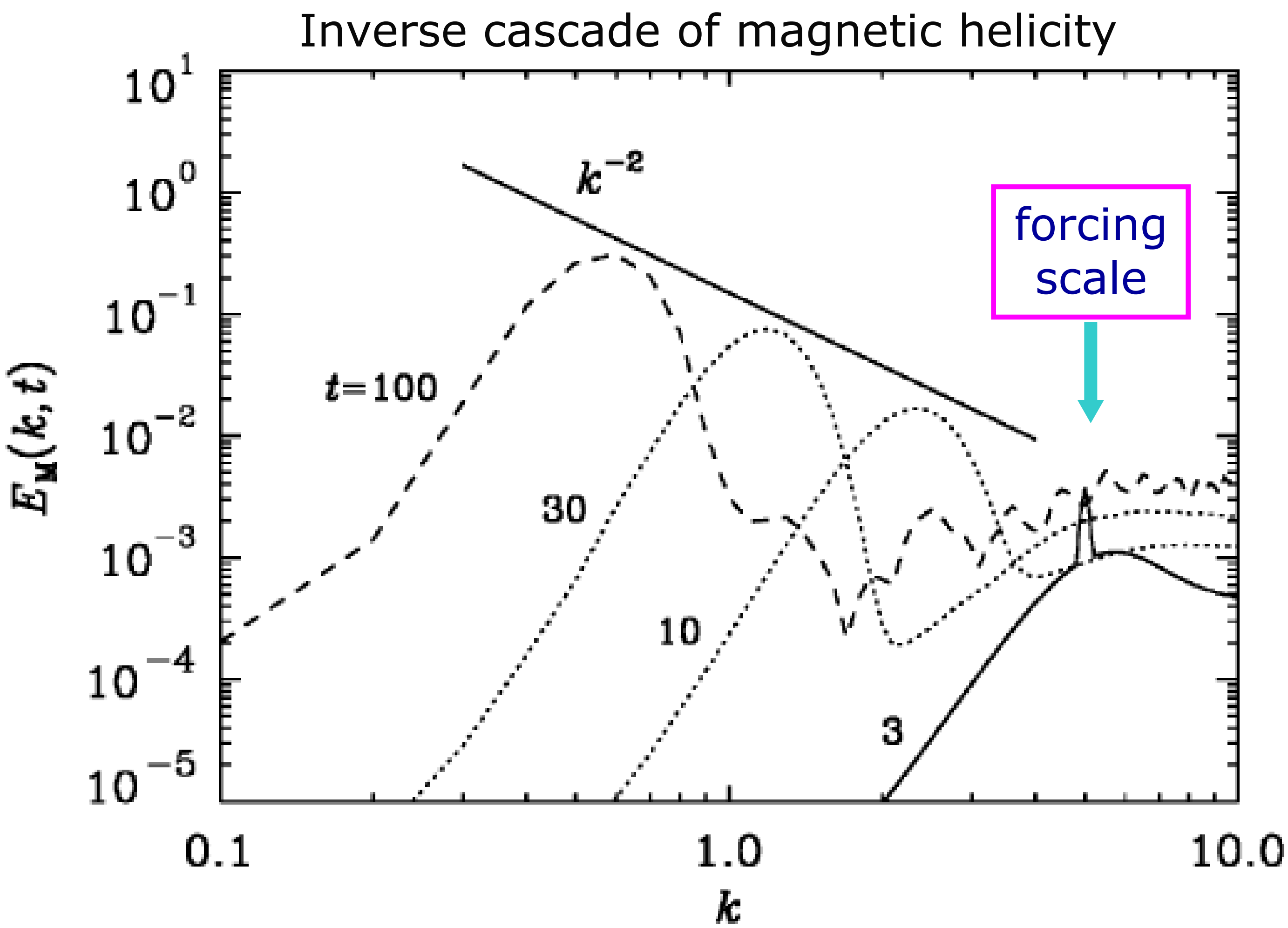


CD – growth of large-scale field on Ohmic timescale (if no shocks)

D – quasi force-free state $B^2 >> b^2$

Large-scale magnetic field growth

Brandenburg, Subramanian, 2005, p.108



<u·ω>≠0 on average over large scale => symmetry breaking

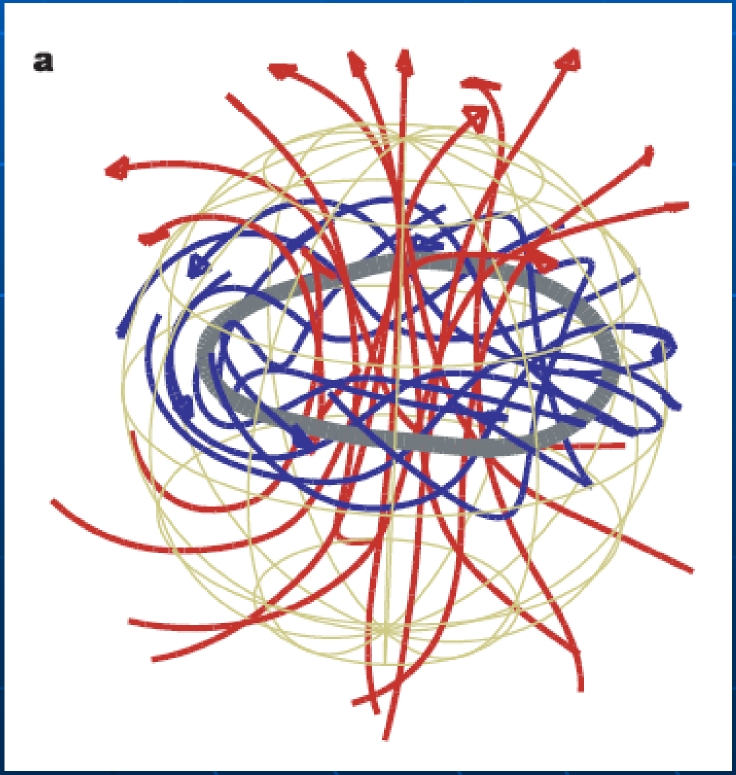
Force-free states

$$\vec{E}_M = \alpha \vec{B} - \beta \vec{J} = 0 \longrightarrow \vec{J} = \frac{\alpha}{\beta}\vec{B}$$

$$\vec{F} = \vec{J} \times \vec{B} = 0$$

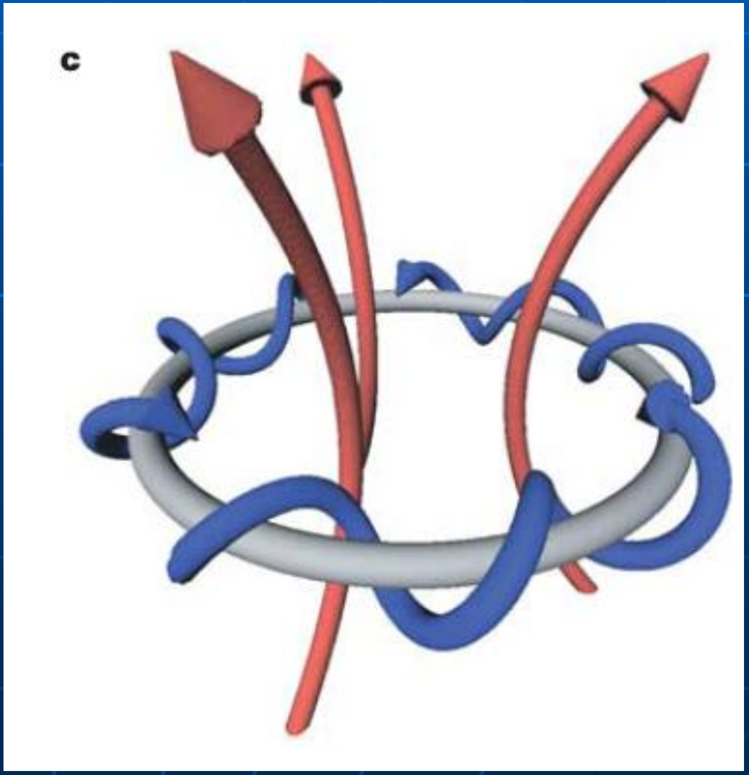
Formation of a star

Braithwaite, Spruit 2004

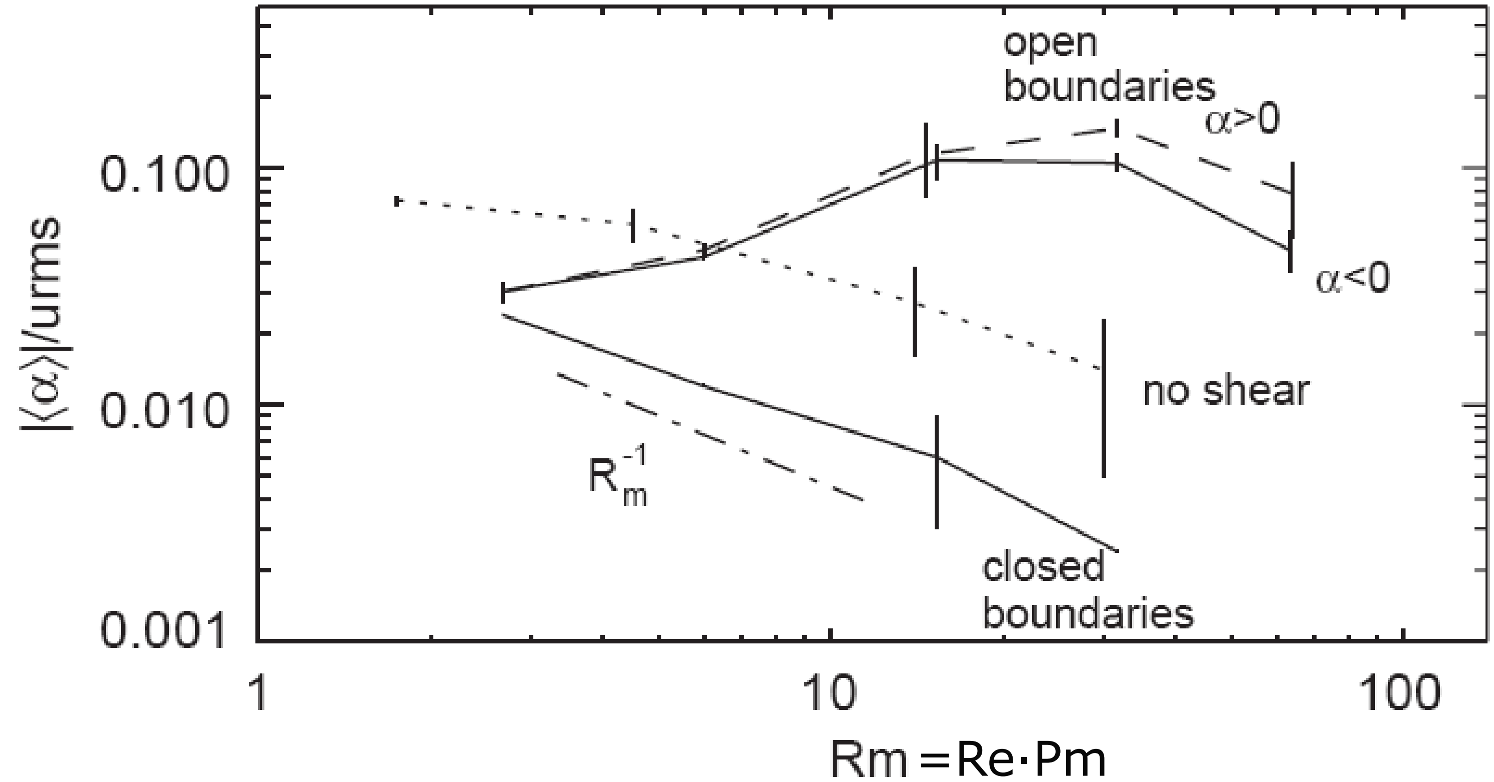


field decay

weak rotation



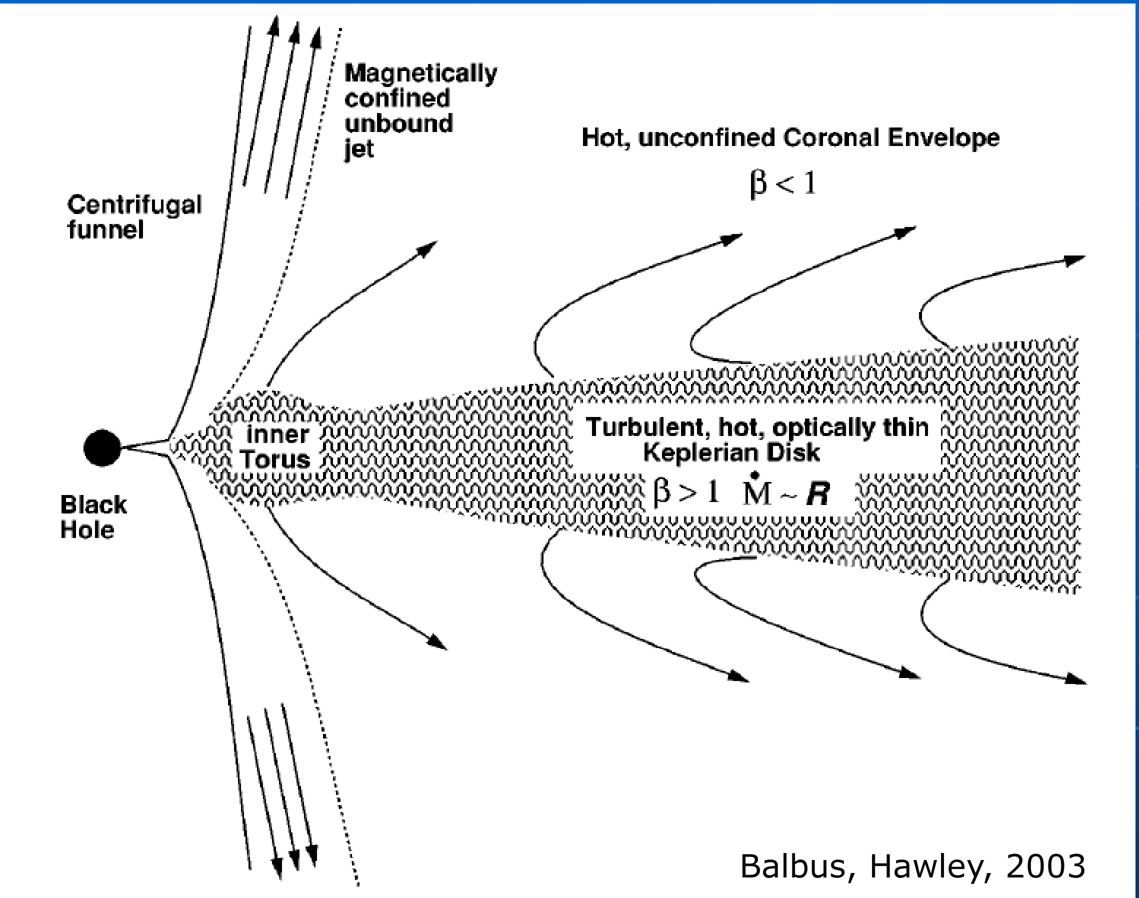
Effect of open boundaries



α–effect is NOT inhibited for open boundaries,
unbalanced outflow of current helicity

Sun? Accretion disks?

Accretion disk dynamo



Balbus, Hawley, 2003

MRI represents Ω-effect

Outflows, convection

Velocity in vertical direction => α-effect

Inverse cascade of magnetic helicity+ advection of field

Large-scale field generation

Model with minimum dynamo action, dynamics, evolution of α, β, γ

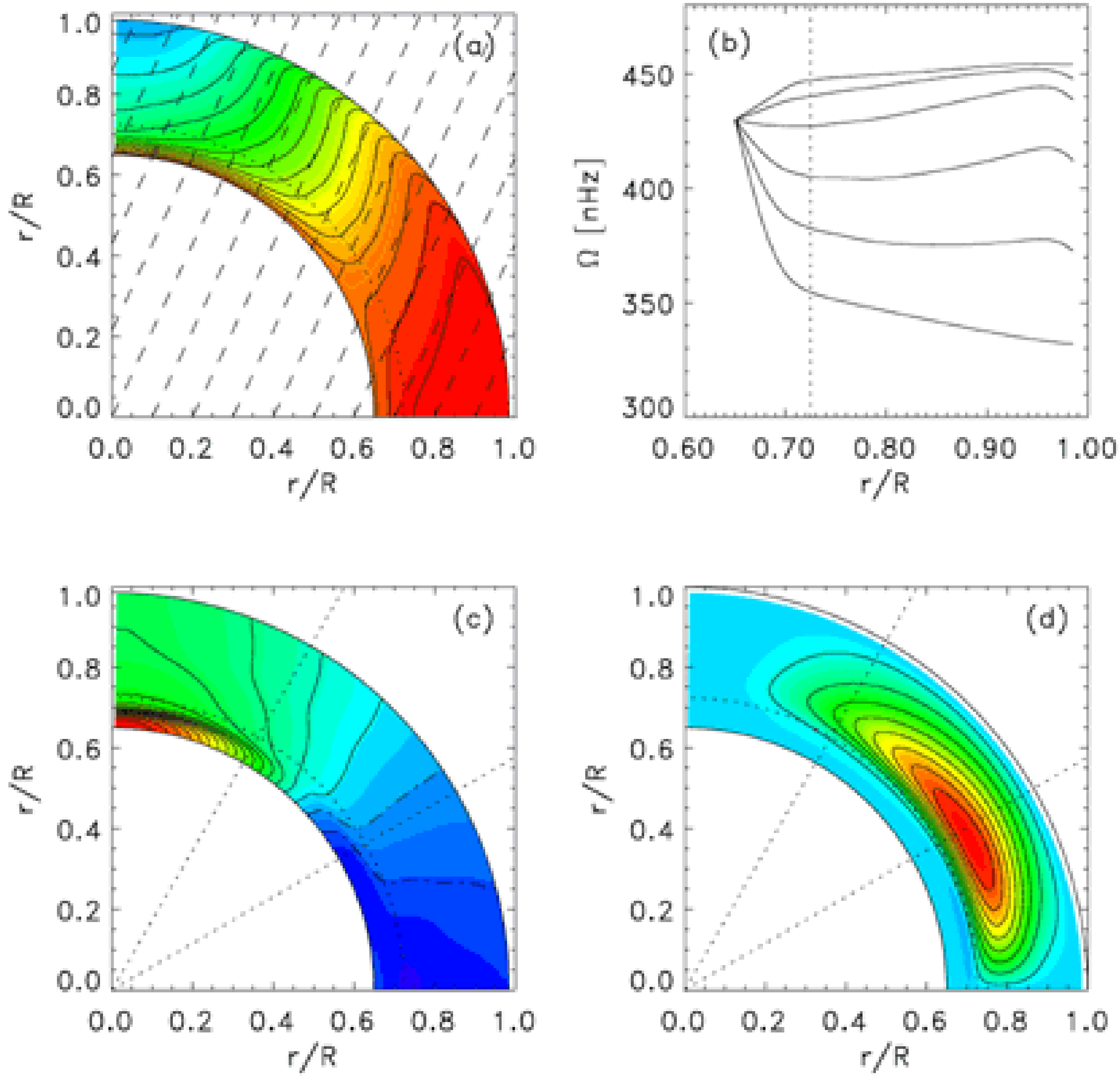
Solution of the problem?

Discussion & Conclusions

- Behavior of dynamical systems strongly depends on non-linear terms => proper non-linear model is essential
- In particular, solutions with outflows/stratification and vertical structure (for disks) have finite α => large-scale field generation
- Proper treatment of boundary conditions and helicity outflows is essential
- Shearing box simulations may not represent the global solution
- One model should be made for accretion & the Sun/Earth, it is easier to test the model on the Sun/Earth

Convection+rotation=>tons of fun

http://www.hao.ucar.edu/Public/research/siv.html



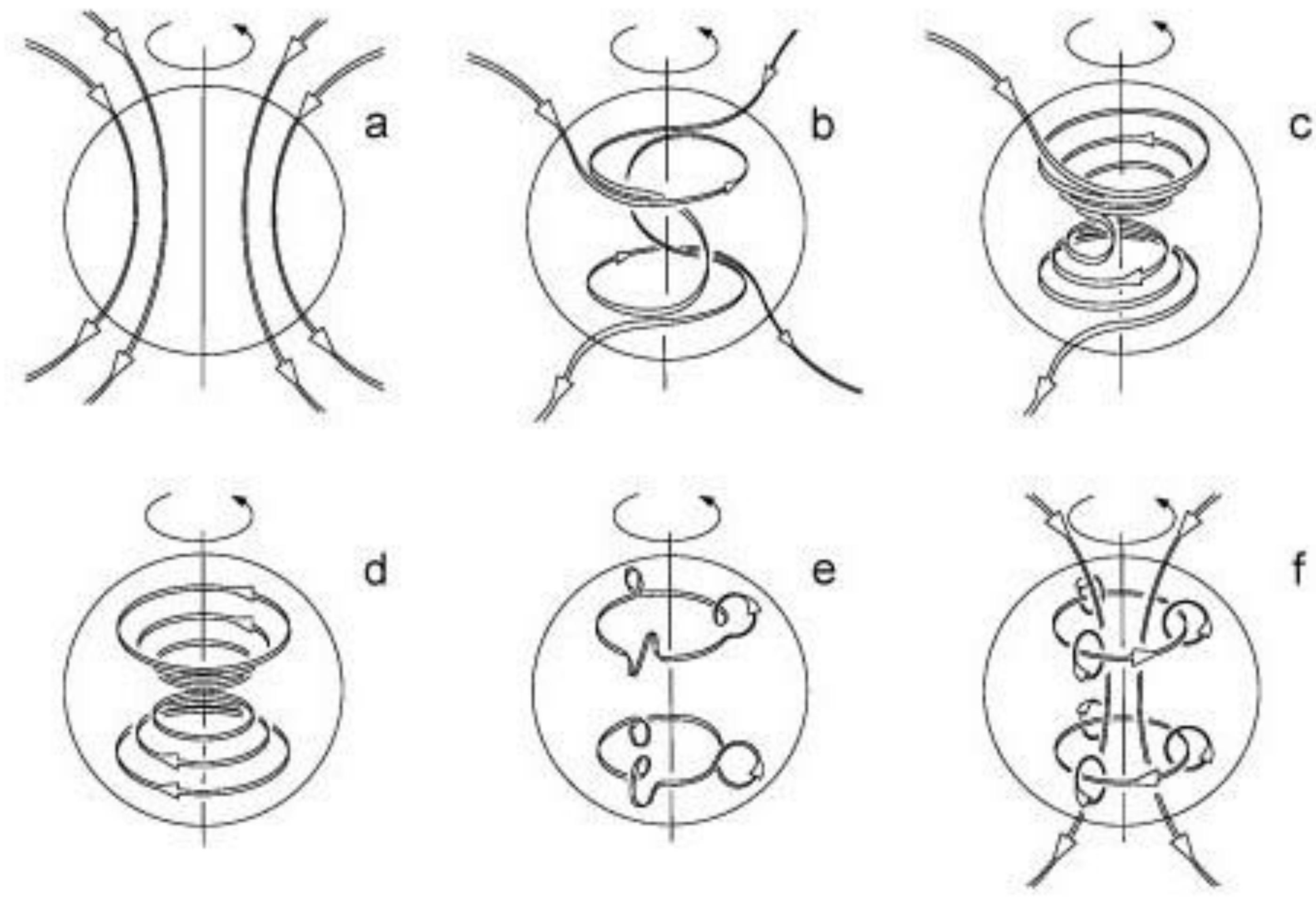
a) Contours of differential rotation, the over-plotted dashed lines indicate a 25 inclination to the axis of rotation as inferred from observations;

b) radial profile of Ω at the latitudes 0°, 15°, 30°, 45°, 60°, and 90°;

c) entropy perturbation required to balance differential rotation;

d) streamlines of meridional flow.

Reversals



Love, J. J., 1999. Astronomy & Geophysics, 40, 6.14-6.19.

http://geomag.usgs.gov/intro.php

Butterfly ☠