EFFECTIVE SEMI-NUMERICAL APPROACH TO BONDI-HOYLE-LITTLETON ACCRETION.

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ABSTRACT. We investigate the properties of an axisymmetric non-magnetized gas flow with and without angular momentum onto a small compact object, in particular, on a Schwarzschild black hole; the velocity of the object itself is assumed to be less than the speed of sound at infinity. We obtain the possible positions of the sound surface position and separatrix.

1. Introduction

We consider axisymmetric one-temperature hydrodynamics.
Does hydrodynamics hold? http://xxx.lanl.gov/abs/astro-ph/9803141

2. General set of problems.

- How many low-luminous stellar objects exist in the Universe? What are the flow patterns for them?
- How does the formation of stars and planetary systems happen?
- Processes in galactic nuclei. Formation of jets.

3. Our problem.

Stationary axisymmetric Bondi-Hoyle accretion. [Bondi, 1952]
What do we want to calculate?

- The position of the sonic surface.
- The position of characteristic surfaces, the boundaries of causally connected regions.
- Behavior of the flow deep inside the sonic surface.
  Progress: http://arxiv.org/abs/astro-ph/0411704.

4. Bondi-Hoyle accretion. Assumptions.

(0) Homogeneous flow at infinity.
(1) Adiabatic law holds. Gas in non-dissipative. Local equation of state (along one flow line):

$$p \sim \rho^{\Gamma}.$$

(2) Gas entropy is a global constant.

$$s = const.$$

Then global equation of state takes the following form:

$$\frac{p}{p_\infty} = \left(\frac{\rho}{\rho_\infty}\right)^{\Gamma}. \tag{1}$$

(3) The projection of angular momentum along the symmetry axis L may not be zero.

(4) The flow is subsonic at infinity

$$\varepsilon = \frac{v_\infty}{c_\infty} < 1.$$

Note: p - pressure, ρ - density, Γ - adiabatic index, v - speed of matter, c - speed of sound, $()_\infty$ - values of the quantities far from the accreting object.

5. System of equations in polar coordinates.

We can rewrite the system of hydrodynamic equations in spherical coordinates (r, θ, ϕ). Symmetry axis is where $\theta = 0$ and $\theta = \pi$. With the assumptions described in previous section, we obtain what follows.

Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{r}(r^2 \rho v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\theta}(\sin\theta \rho v_\theta) = 0, \tag{2}$$

Radial part of Euler equation

$$\frac{\partial(\rho v_r)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2(\rho v_r^2 + p)) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\rho v_r v_\theta) = 2\frac{p}{r} - \frac{r_g c^2 \rho}{2r^2} + \frac{\rho v_\theta^2}{r}, \tag{3}$$

Component of Euler equation along $\mathbf{e}_\theta$ vector

$$\frac{\partial(\rho v_\theta)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_\theta v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta(\rho v_\theta^2 + p)) = \frac{p}{r}\cot\theta - \frac{\rho v_r v_\theta}{r}, \tag{4}$$

6. Equations.

Consider stationary case.

- Conservation of matter

$$div(\rho\vec{v}) = 0 \tag{5}$$

- Circulation theorem

$$\oint \vec{v} d\vec{l} = const \tag{6}$$

- Bernoulli integral

$$w + \phi_{\mathrm{g}} + \frac{\mathbf{v}^2}{2} = const \tag{7}$$

The system (5), (6), (7) is the same that the system (2), (3), (4). Write the former system in polar coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\rho v_\theta) = 0 \tag{5'}$$

$$\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial}{\partial\theta}(v_r) = \frac{L}{r} \tag{6'}$$

$$\frac{c_\infty^2 \rho^{\Gamma-1}}{(\Gamma-1)\rho_\infty^{\Gamma-1}} - \frac{c^2 r_g}{2r} + \frac{1}{2}\left(v_r^2 + v_\theta^2 + L^2/r^2\right) = En \tag{7'}$$

7. Transformation of equations.

From now on $\mathbf{L}=\mathbf{0}$. Let us notice equation (5) has a potential form as well as equation (6). So, we can define the function Φ called potential for every of these equations to satisfy them automatically.

People usually introduce the potential for equation (6), killing therefore continuity equation. The potential is :

$$\rho\mathbf{v} = \nabla\Phi \times \mathbf{e}_\phi / 2\pi r \sin\theta,$$

and the method used has a name "Grad-Shafranov approach".

However, to write down the potential for equation (5) is more convenient for our purposes. We get, accordingly: $\mathbf{v} = \nabla\Phi$ or componentwise:

$$\frac{\partial}{\partial r}\Phi = v_r, \quad \frac{\partial}{\partial \theta}\Phi = r v_\theta. \tag{8}$$

We will use this potential in further calculations.

Notice, the potential for homogeneous flow with speed v is $\Phi = -vr\cos(\theta)$, what is simpler than potential $\Phi \sim r^2 \sin(\theta)^2$ in Grad-Shafranov approach.

From (7) and (4) we obtain for matter density:

$$\rho = \rho_\infty \left(\frac{\Gamma-1}{c_\infty^2}\right)^{\frac{1}{\Gamma-1}} \left(-\frac{r_g c^2}{2r} + \frac{1}{2}(\Phi_r^2 + \Phi_\theta^2/r^2) - En\right)^{\frac{1}{\Gamma-1}} \tag{9}$$

Then we substitute this density in (6) and finally get the following equation:

$$\begin{gathered}
-(\Gamma-1)\cot(\theta)\Phi_\theta^3 - \left((\Gamma+1)\Phi_{\theta\theta} + r\left(2(\Gamma-2)\Phi_r + r(\Gamma-1)\Phi_{rr}\right)\right)\Phi_\theta^2 + \\
+ r\left((\Gamma-1)\cot(\theta)\left(r_g c^2 - r\Phi_r^2 + 2rEn\right) - 4r\Phi_r\Phi_{r\theta}\right)\Phi_\theta + \\
+ r\left((\Gamma-1)\Phi_{\theta\theta}\left(r_g c^2 - r\Phi_r^2 + 2rEn\right) + r\left(\Phi_r\left(r_g(2\Gamma-3)c^2 - 2r(\Gamma-1)\Phi_r^2 + 4rEn(\Gamma-1)\right)\right)\right) + \\
+ r^2\left(\left(r_g c^2 + 2rEn\right)(\Gamma-1) - r(\Gamma+1)\Phi_r^2\right)\Phi_{rr} = 0.
\end{gathered} \tag{10}$$

in partial derivatives of the second order, quasi linear in them.

Equation (10) has narrower domain of applicability than Grad-Shafranov system. Nonetheless, we can derive one equation, which fully describes hydrodynamics, in former approach.

Note: Grad-Shafranov approach also admits derivation of one equation for its potential only [Ruffert, 1997].

8. Solution method.

Equation (9) contains integer powers of Φ only, therefore we can expand it in series in Legendre polynomials of θ.

We assume the solution for potential exists in the form

$$\Phi(r,\theta) = \sum_{i=0}^{\infty} \varepsilon^i \sum_{l=0}^{\infty} P_l(\cos\theta) f_l(r) \tag{11}$$

Series (11) doesn't contain terms with such i and l that $i + l \bmod 2 = 1$ due to symmetry of the flow under parameters change $\theta \to \pi - \theta, \varepsilon \to -\varepsilon \Rightarrow \Phi \to \Phi$.

We approximate the solution by the finite sum

$$\Phi(r,\theta) = \sum_{i=0}^{n} \varepsilon^i \sum_{l=0}^{N} P_{2l+(i) mod 2}(\cos\theta) f_l(r) \tag{12}$$

Where

$$P_l(\cos\theta) = (1, \cos\theta, (3\cos^2\theta - 1)/2, ...)$$

are Legendre polynomials.

It was mentioned already that gas flow has a potential $\Phi = -vr\cos(\theta)$ at infinity, that contains only first Legendre polynomial. So we might guess the weights of higher polynomials are low, (12) approaches (11).

9. Results. General properties of the flow.

Sound surface is can be defined is a surface where

$$(\Gamma-1)(2En+\frac{r_g c^2}{r})=(\Phi_r^2+\frac{\Phi_\theta^2}{r^2})(\Gamma+1). \tag{13}$$

The regions of parabolicity of (9) are determined by the same equation (13). Equation (9) is hyperbolic in regions, closer to the center than sonic surface and elliptic in farther regions.

10. Pictures.

We make calculations for $n=3$, $N=2$. The error is less than approximately 20% for $\varepsilon<0.5$.

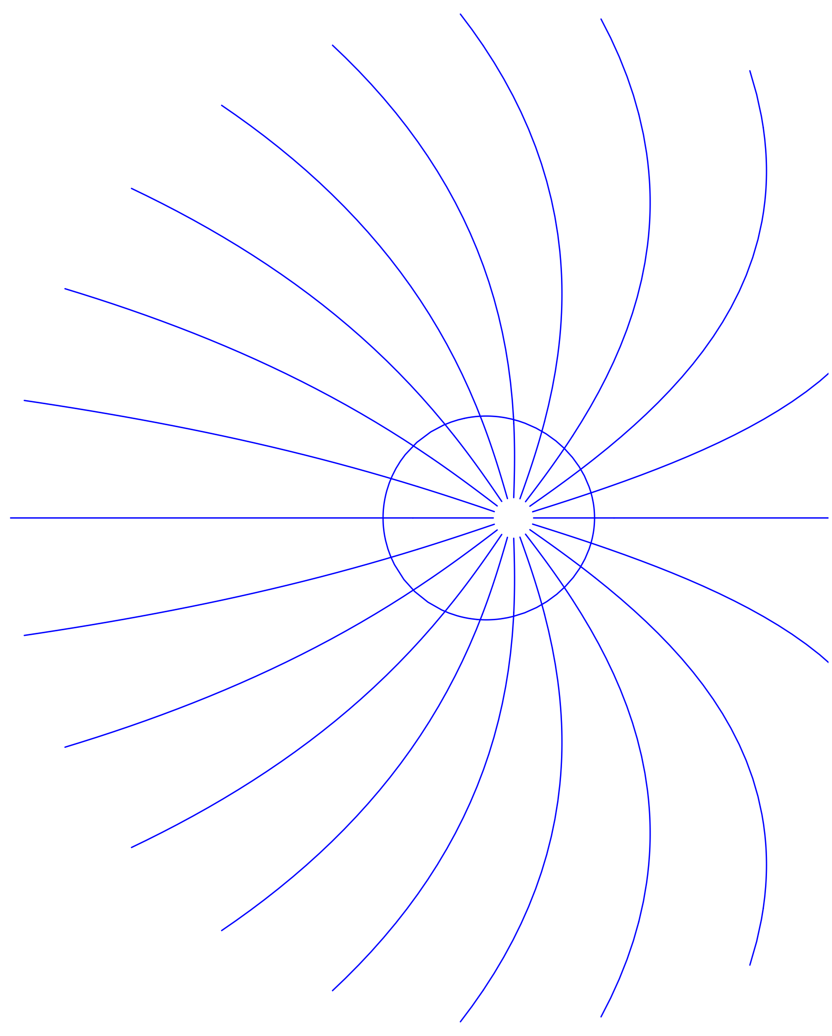

Figure 1. Flow lines and sonic surface. $\varepsilon=0.5$, $\Gamma=3/2$. Gas flows from right to left.

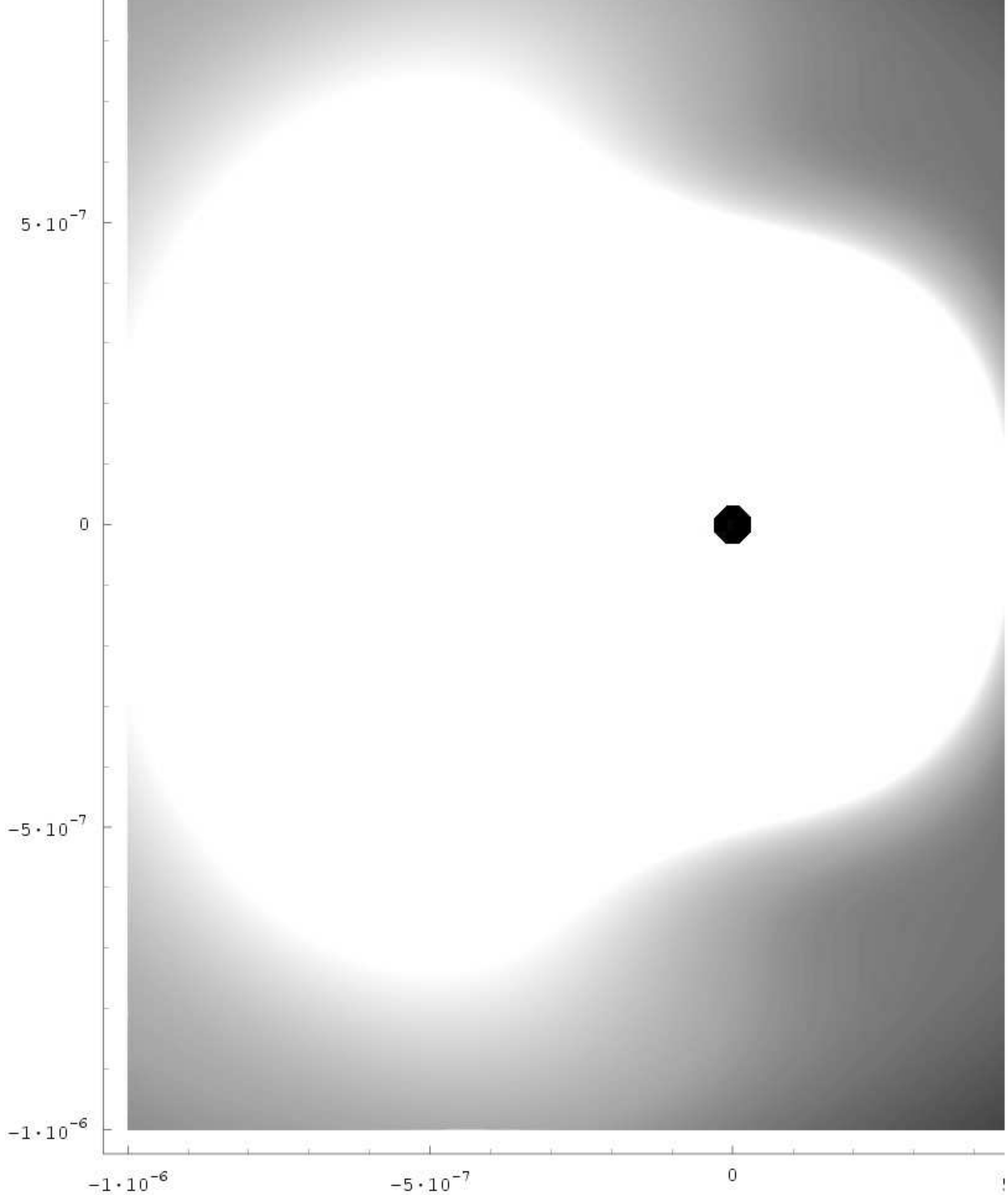

FIGURE 2. The intensity of energy release due to bremsstrahlung. $\varepsilon = 0.5$, $\Gamma = 3/2$. Black dot is the accreting object. axis x, y - dimensionless distance, Bondi radius is equal to 1/16.

11. Conclusion.

The method used allows us to make calculations in general axisymmetric case and doesn't demand lots of computer time (which is essential).

The result of this study confirms the previous work [Shcherbakov, 2004]. The assumption of constant $\Gamma < 5/3$ in hydrodynamic approach is weak. The assumption of the absence of turbulence is weak. But we cannot leave these assumptions in our technique. So, we will obtain only some approximation that may not be realistic. The efficiency of accretion is believed to be less than that in Bondi accretion for sources with non-disk accretion [Narayan,2002], but we obtained the increase of efficiency.

The future plans:

- Study the case with non-zero angular momentum projection L
- Calculation of the flow patterns in 2-temperature hydrodynamics