Magnetized spherically symmetric accretion flows

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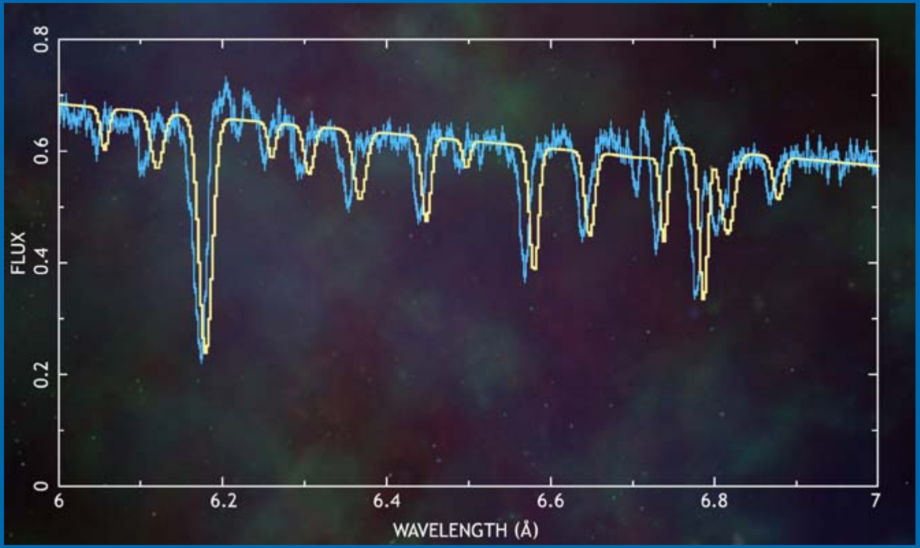
What do we want to know?

How astrophysical flows work

Credit: NASA/Dana Berry

then we can
predict radiation spectrum



Credit: NASA/CXC/U.Michigan/J.Miller et al

Laws of turbulence
(fusion reactors, oceans,
Earth atmosphere etc.)

Ways to approach

	Numerical simulations	Closure models
Essence	$3\,D$ equations with shock waves on $\vec{v}, \vec{B}, \rho, p, T$	$\geq 3\,D$ smooth equations on averages and correlations
Computational power	N^4	N^3 as maximum N^2 or N^1 for symmetric

We believe in equations vs There is no complete theory

☺ ☹

Perspective

- Numerical simulations

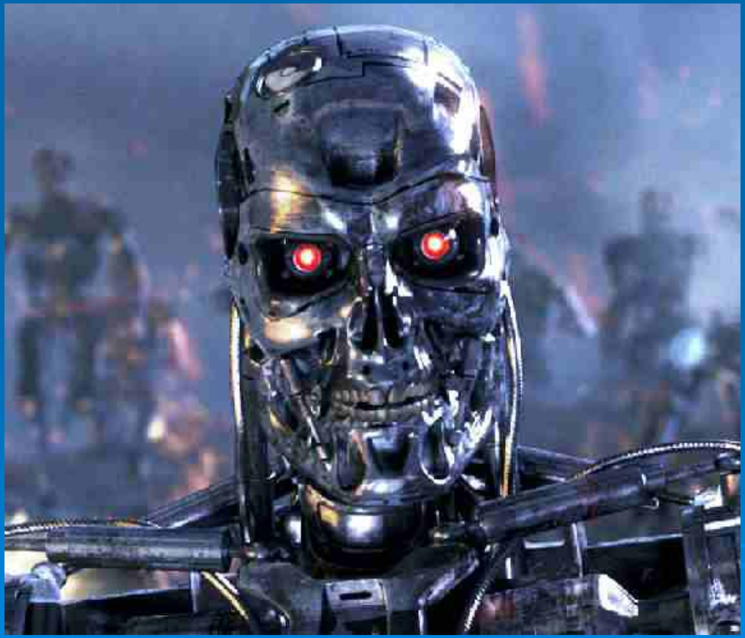
More powerful computers

- Closure theories

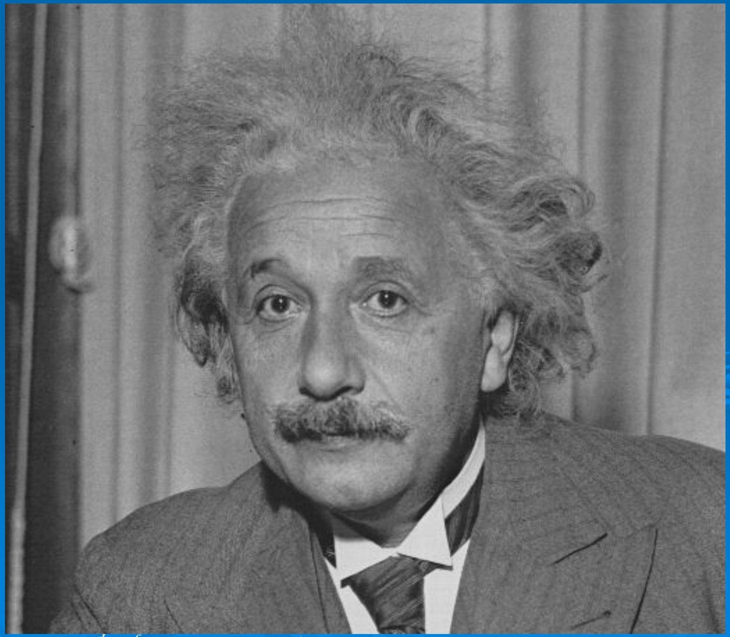
More powerful brains

Don’t let machines take over humans

We can do this

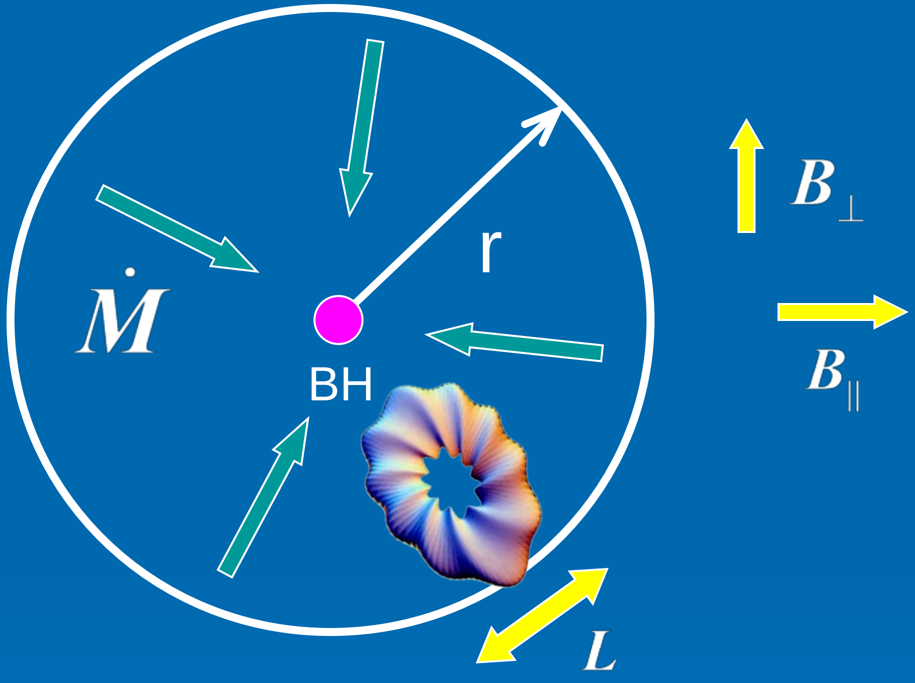


vs



Spherically symmetric closure

$$\rho, T, \vec{v}$$



v = regular (V_{in}) + isotropic turbulent (u) velocities

ρ – density

T – temperature

$B_\perp$ – perpendicular magnetic field

$B_\parallel$ – radial magnetic field

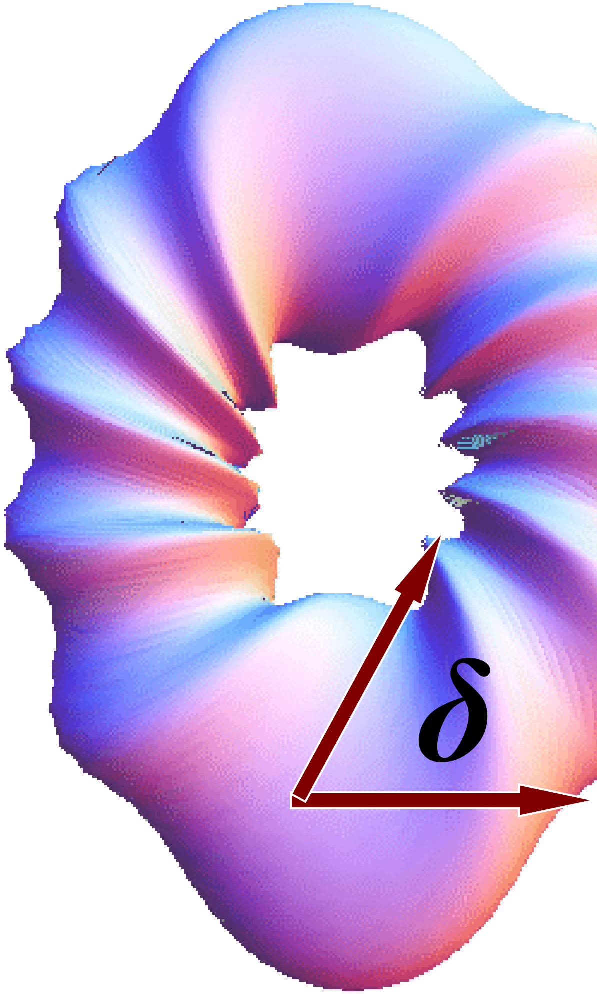
L – characteristic length scale

δ – magnetic helicity

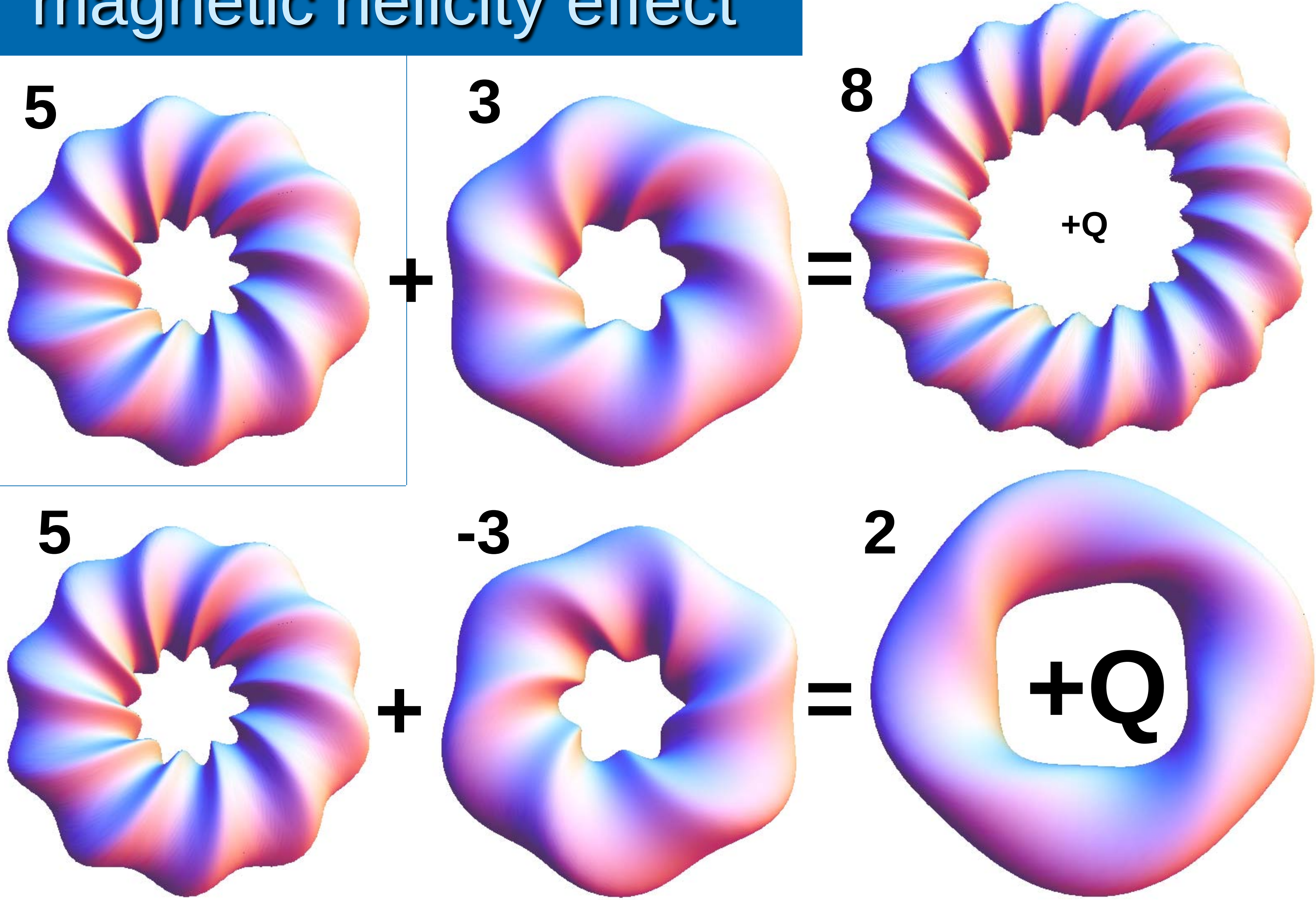
8 functions of radius
8 equations needed

Flux tubes

δ – winding angle



Interactions of flux tubes magnetic helicity effect



5 + 3 = 8 +Q

5 + -3 = 2 +Q

How to include dissipation?

Magnetic field inhibits accretion,

but **how much is the effect**?

No dissipation → No matter inflow

No theory available → Use other (simple) numerical simulations

The better we correspond to experiment, the more reliable the result is.

Dissipation of hydrodynamic turbulence	Dissipation of MHD turbulence	Dynamo action
Sreenivasan, 1995	Biskamp, 2003	Shchekochihin,2004

+ phenomenological helicity conservation

Equations

1D (radial) equations on $V_{in}, \rho, T, B_\perp^2, B_\parallel^2, u^2, L, \delta$

$\rho V_{in} r^2 = const$ $\dot{M}$ mass flux (+1)

momentum and energy fluxes (+2)

Angular averaged ideal MHD

$$V_{in}V'_{in} + \frac{r_g c^2}{2(r-r_g)^2} + \frac{R}{\mu}\frac{(\rho T)'}{\rho} + \frac{(\rho u^2)'}{3\rho} + \frac{(r^2 B_\perp^2)'}{4\pi\rho r^2} - \frac{(r^4 B_\parallel^2)'}{8\pi\rho r^4} = 0$$

$$V_{in} \ldots_{in} + W + \frac{r_g c^2}{2(r-r_g)^2} + 4V_{A\perp}V_{A\perp}' = 0$$

$L = const$ (at $r >> r_B$)

$L = \gamma r$ (at $r << r_B$)

and

$$L = \gamma r\left(\exp\left(-\frac{r}{r_B}\right) + \frac{r_B}{r}\left(1 - \exp\left(-\frac{r}{r_B}\right)\right)\right)$$

Magnetic helicity conservation

$$\tan\delta = \tan\delta_\infty \frac{L_\infty}{L}\frac{V_{A\parallel\infty}^2 + 2V_{A\perp\infty}^2}{V_{A\parallel}^2 + 2V_{A\perp}^2}$$

length scale and winding angle (+2)

$$(B_\parallel^2)' = -4\frac{B_\parallel^2}{r} + \frac{c_{BB}B_\parallel^2 V_{A\parallel}\exp(-\tan\delta) - c_{Bu}(B_\parallel^2 + 2B_\perp^2)u}{V_{in}L} - \frac{\rho c_{11}}{V_{in}L_\infty}\left(\frac{\beta R T_\infty}{\mu}\right)^{3/2}$$

$$(B_\perp^2)' = -2B_\perp^2\frac{(V_{in}r)'}{V_{in}r} + \frac{c_{BB}B_\perp^2 V_{A\perp}\exp(-\tan\delta) - c_{Bu}(B_\parallel^2 + 2B_\perp^2)u}{V_{in}L} - \frac{\rho c_{22}}{V_{in}L_\infty}\left(\frac{\beta R T_\infty}{\mu}\right)^{3/2}$$

$$(u^2)' = -\frac{2}{3}\frac{2u^2}{r} + \frac{\ldots - c_{uB}(V_{A\parallel}^3 + 2V_{A\perp}^3)\exp(-\tan\delta)}{V_{in}L} - \frac{\rho c_{00}}{V_{in}L_\infty}\left(\frac{\beta R T_\infty}{\mu}\right)^{3/2}$$

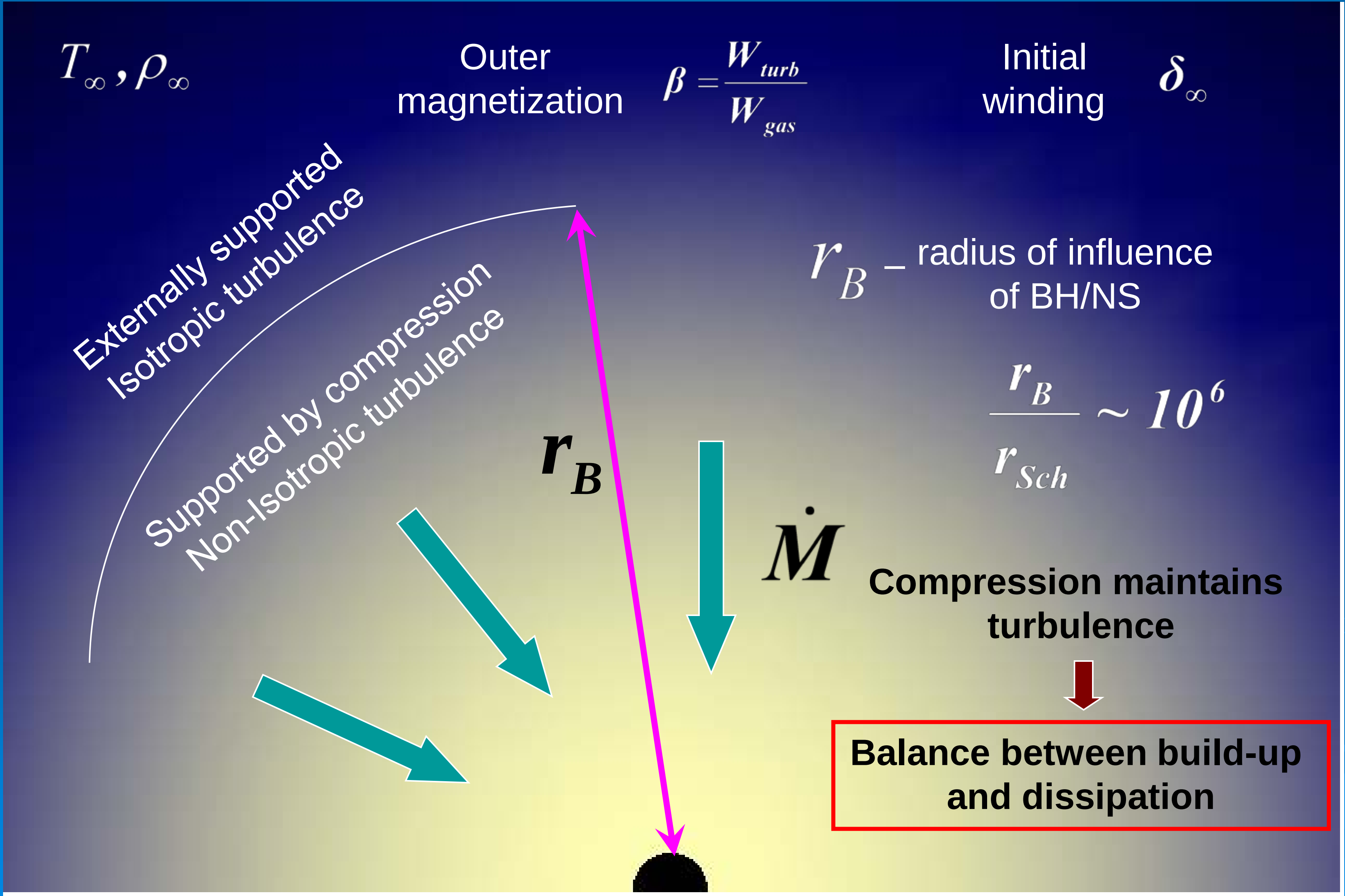
- ➢Small external isotropic energy input,
- ➢Freezing-in amplification of turbulence
- ➢Empiric energy dissipation
- ➢Effects listed above are summed for squares

evolution of turbulence (+3),

$$(\ldots)' \Leftrightarrow \frac{d(\ldots)}{dr}$$

Search for maximum $\dot{M}$

We don’t care about substructure



T_∞, ρ_∞

Outer magnetization $\beta = \frac{W_{turb}}{W_{gas}}$

Initial winding δ_∞

Externally supported
Isotropic turbulence

Supported by compression
Non-Isotropic turbulence

r_B

r_B – radius of influence of BH/NS

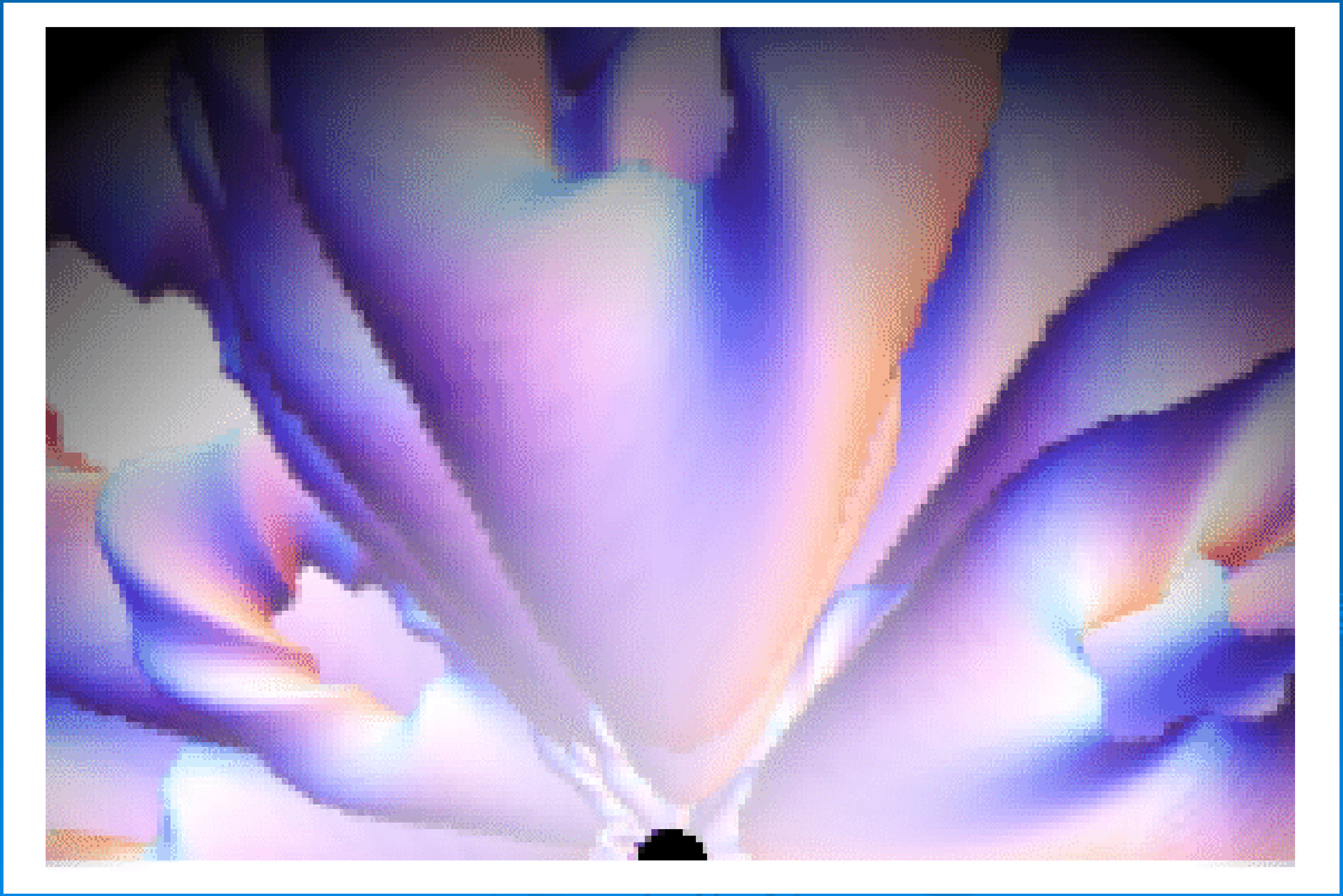
$$\frac{r_B}{r_{Sch}} \sim 10^6$$

$\dot{M}$

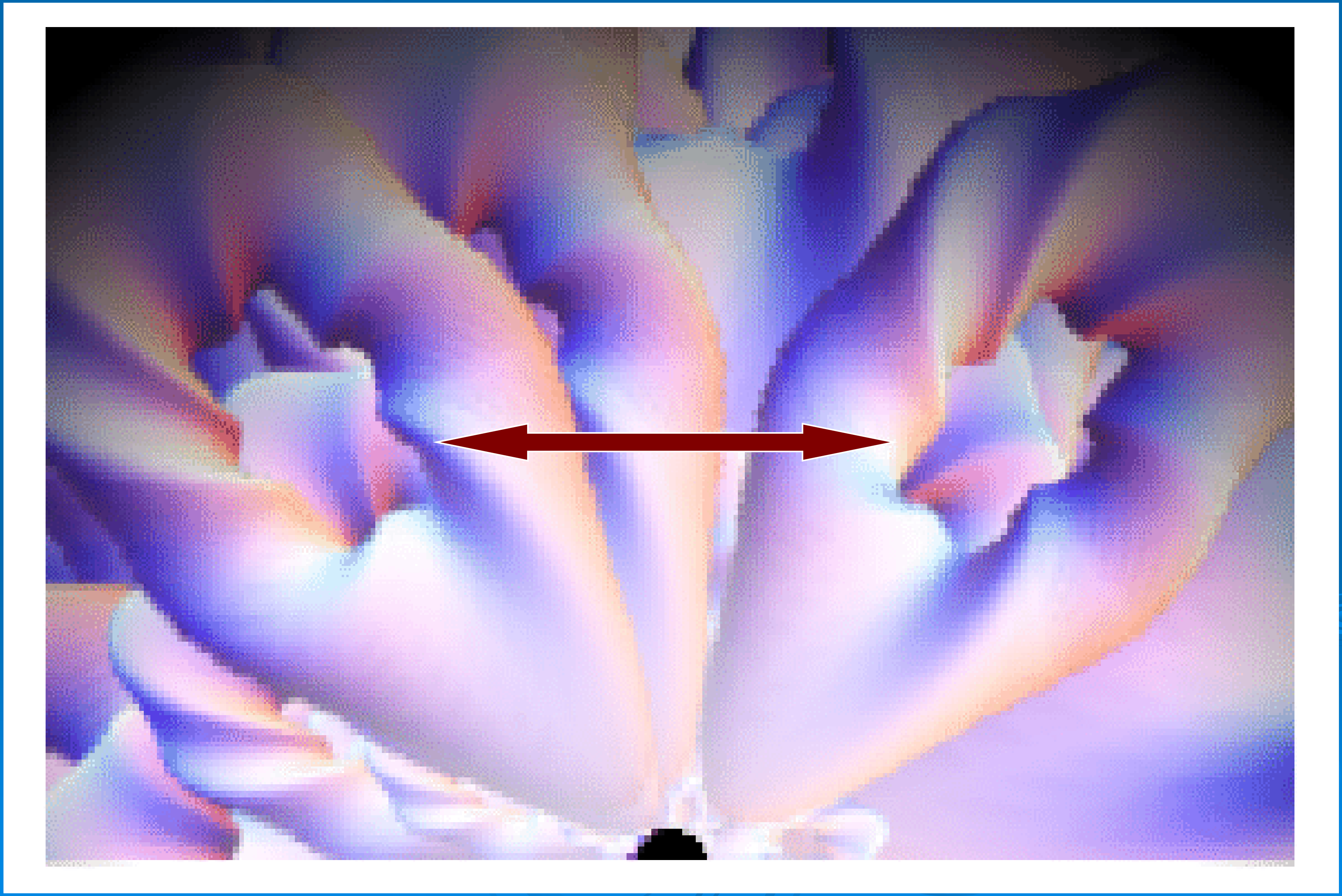
Compression maintains turbulence

Balance between build-up and dissipation

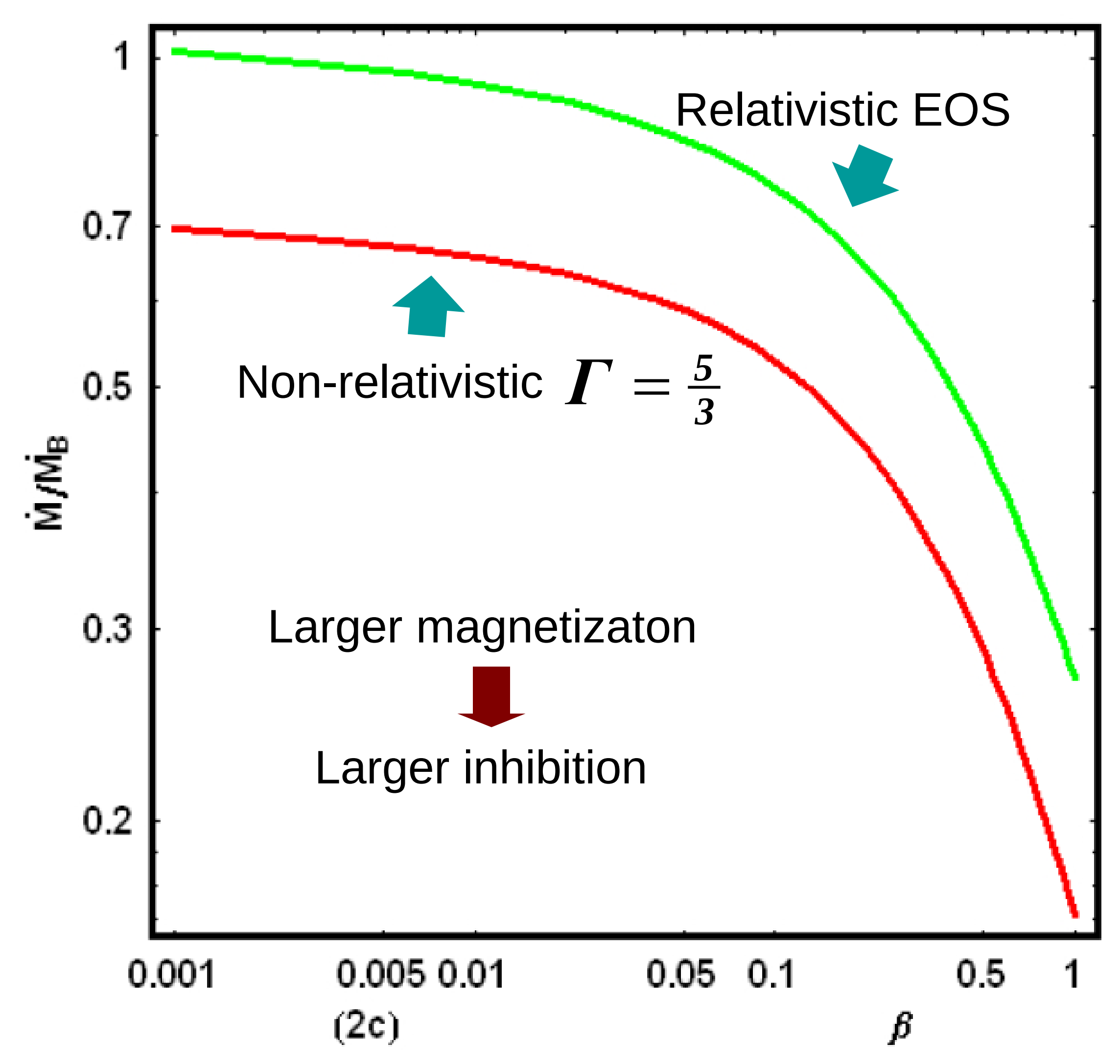
Flux tube accretion, no diffusion



Flux tube accretion, perpendicular diffusion



Results. Accretion rate



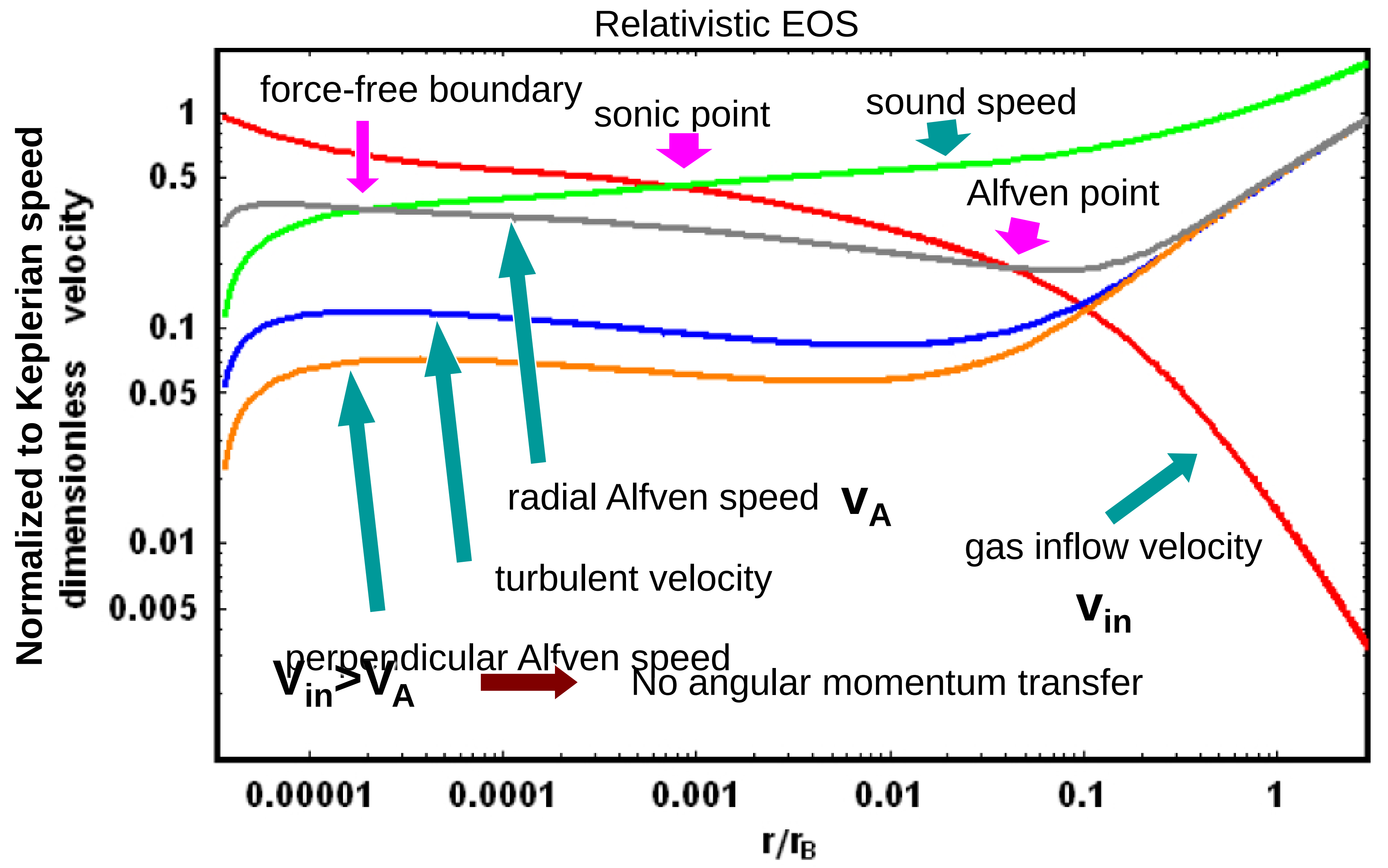
Relativistic EOS

Non-relativistic $\Gamma = \frac{5}{3}$

Larger magnetizaton

Larger inhibition

Results. Velocities



force-free boundary

sonic point

sound speed

Alfven point

radial Alfven speed v_A

turbulent velocity

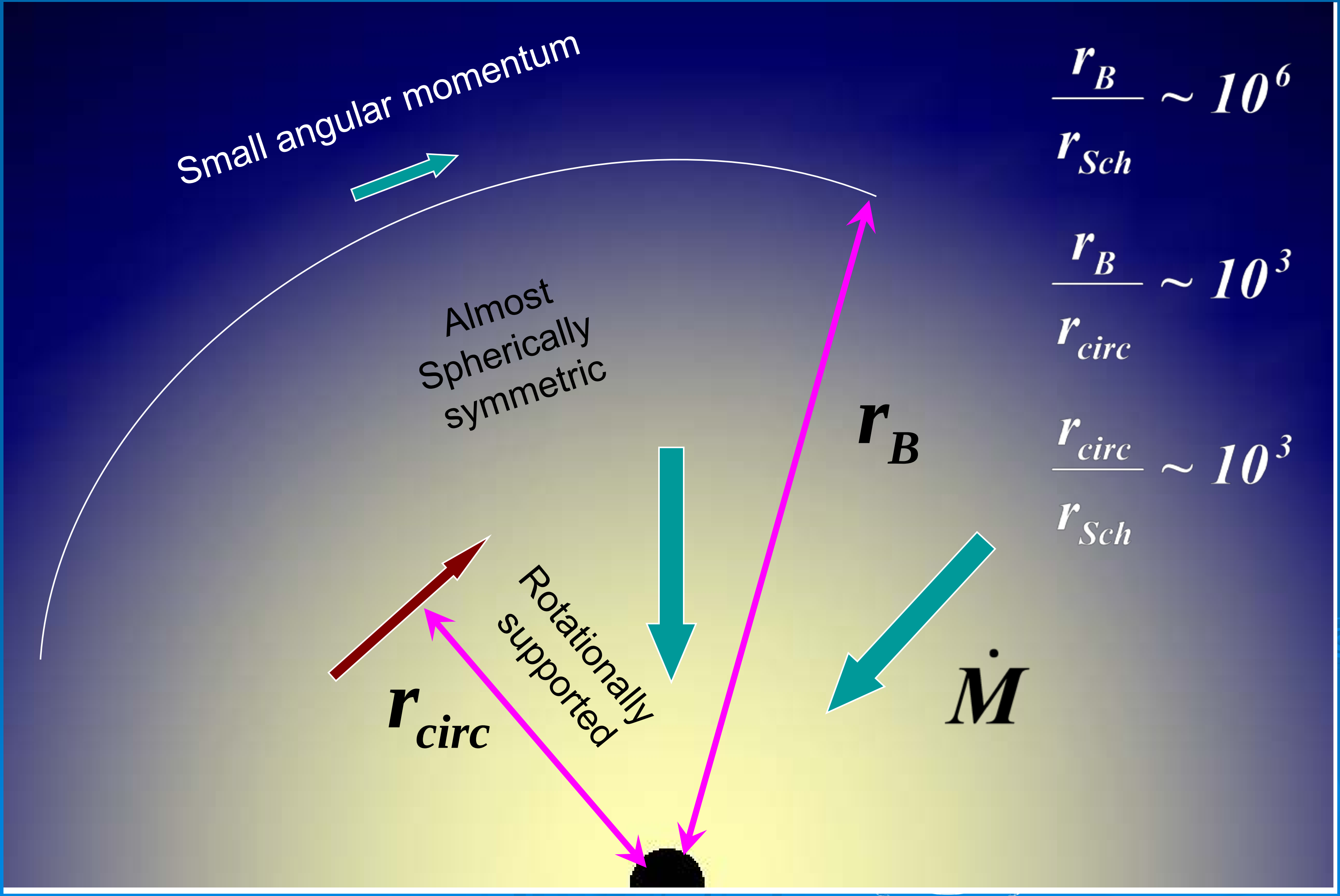
gas inflow velocity

v_{in}

perpendicular Alfven speed

$V_{in}>V_A$ → No angular momentum transfer

Case with angular momentum

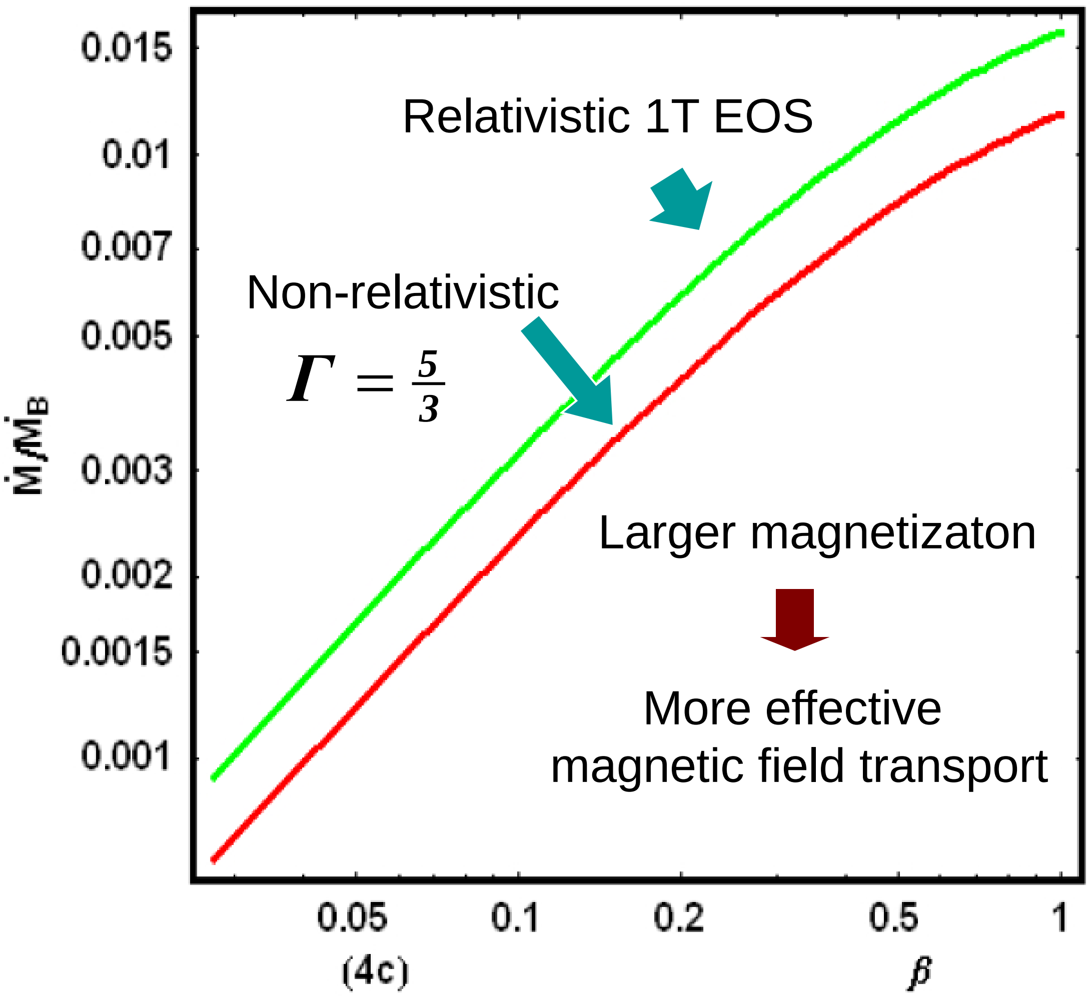


$$\frac{r_B}{r_{Sch}} \sim 10^6$$

$$\frac{r_B}{r_{circ}} \sim 10^3$$

$$\frac{r_{circ}}{r_{Sch}} \sim 10^3$$

Non-zero angular momentum. Accretion rate



Relativistic 1T EOS

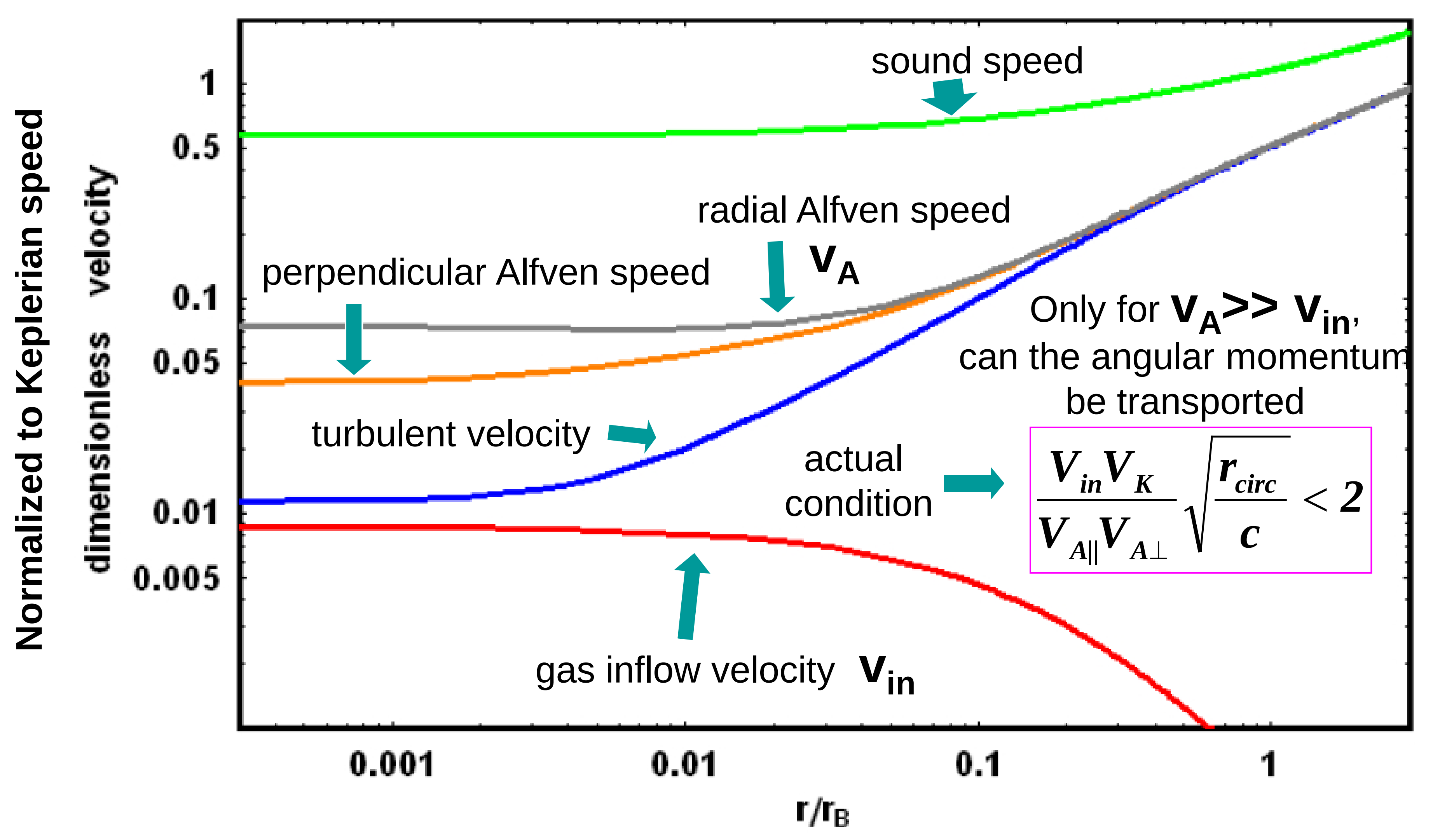
Non-relativistic

$$\Gamma = \frac{5}{3}$$

Larger magnetizaton

More effective magnetic field transport

Non-zero angular momentum. Velocities



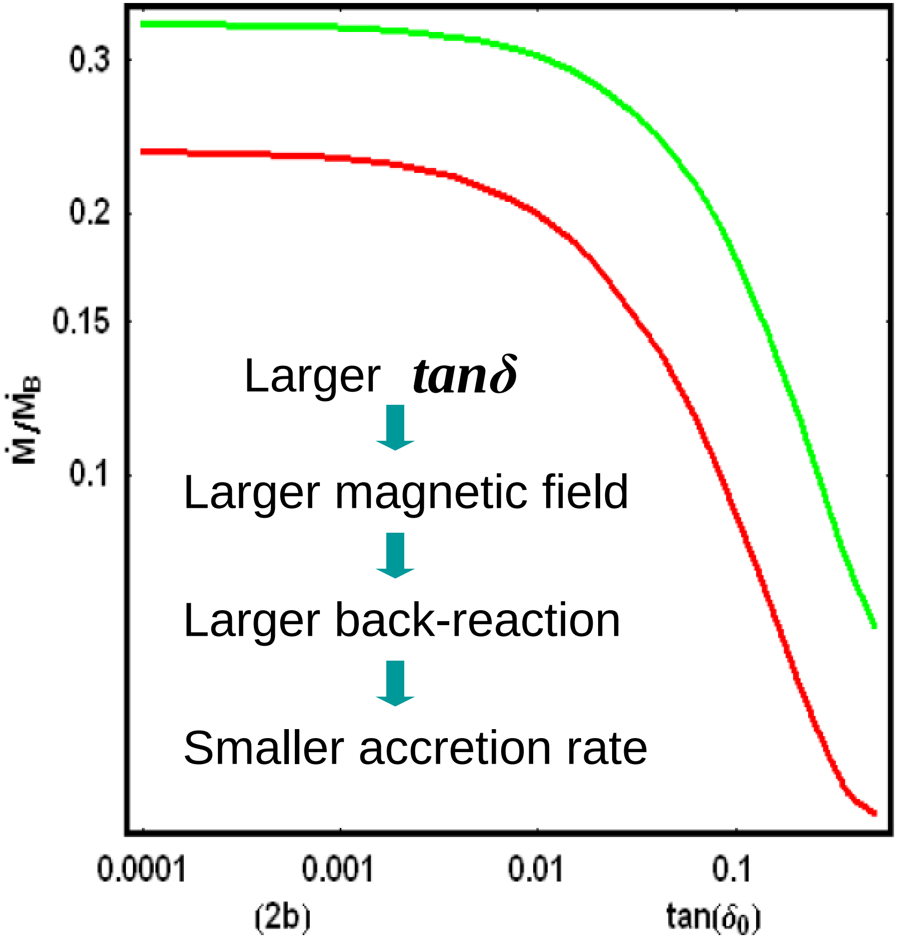
Only for **$v_A >> v_{in}$**, can the angular momentum be transported

actual condition →

$$\frac{V_{in}V_K}{V_{A\|}V_{A\perp}}\sqrt{\frac{r_{circ}}{c}} < 2$$

Accretion rate vs. winding angle

No angular momentum



Larger *tanδ*

↓

Larger magnetic field

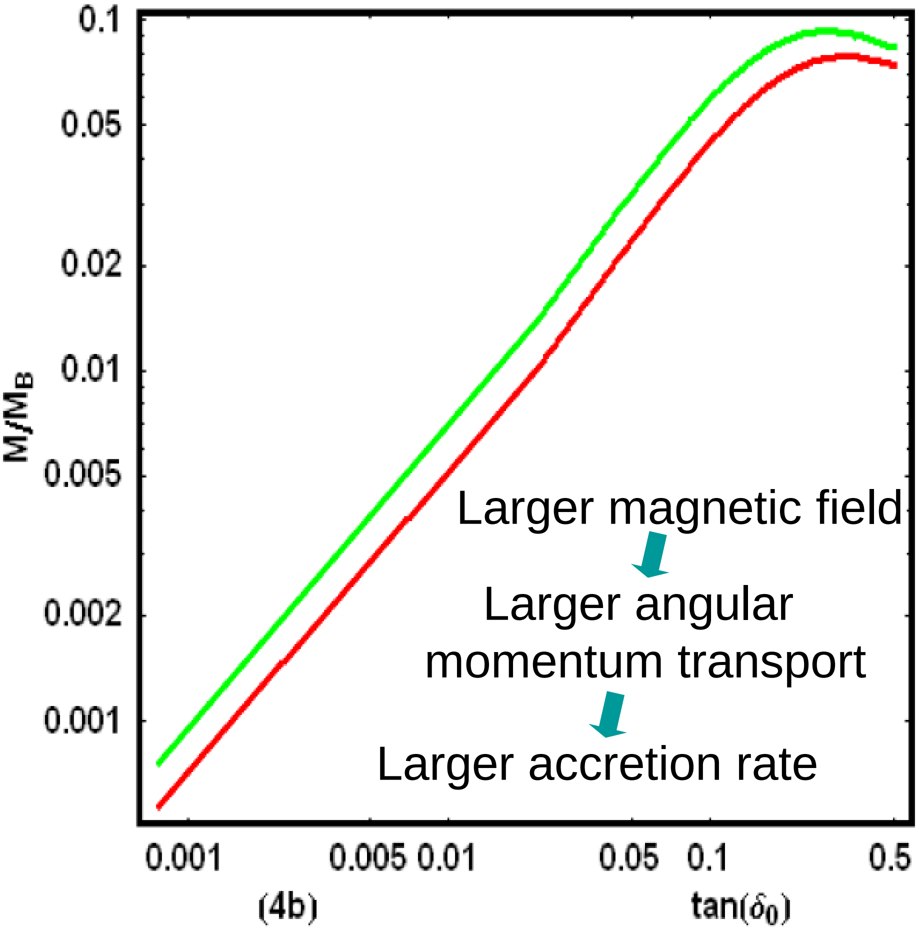
↓

Larger back-reaction

↓

Smaller accretion rate

With angular momentum $r_{circ} = 10^3\, r_{Sch}$



Larger magnetic field

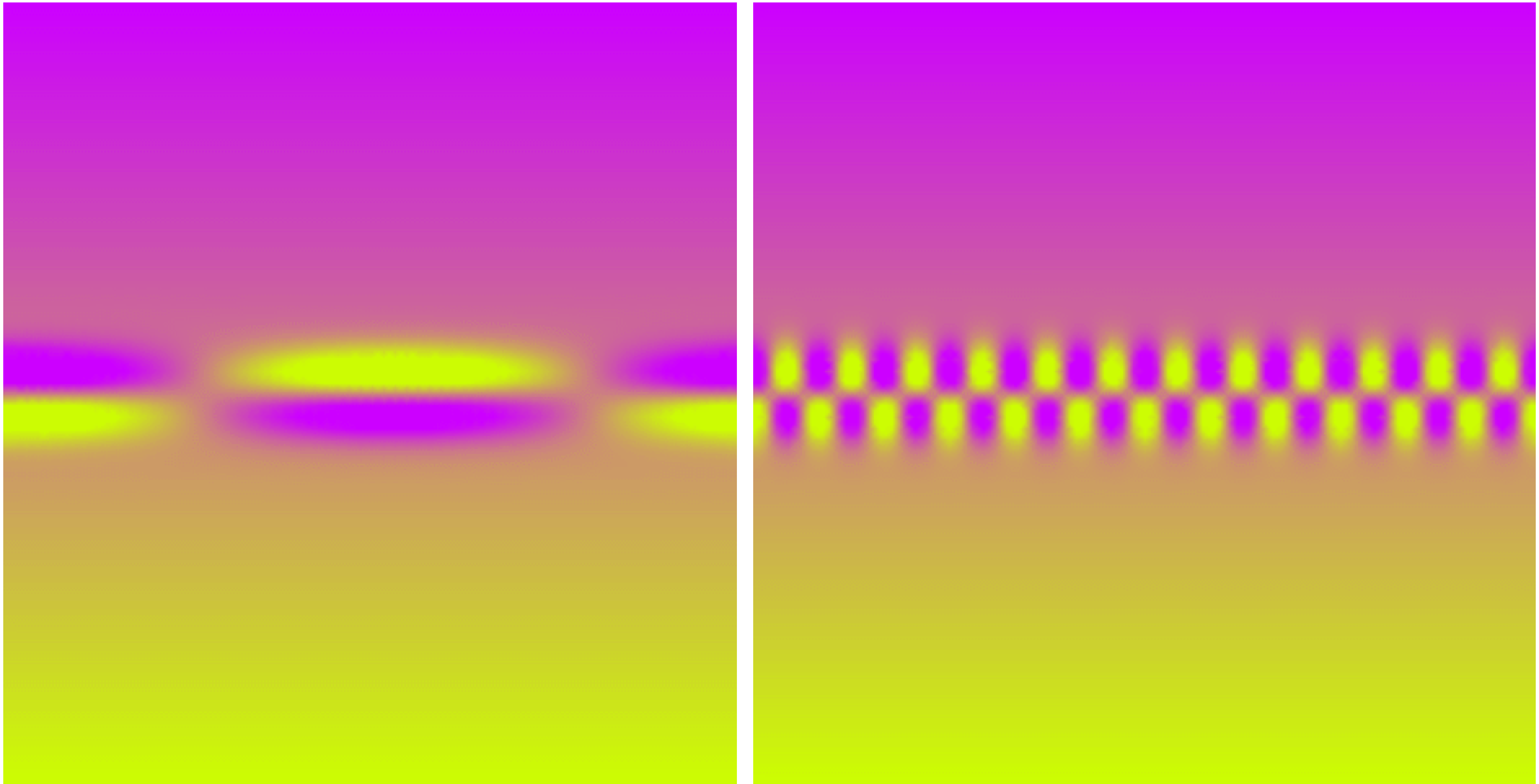
↓

Larger angular momentum transport

↓

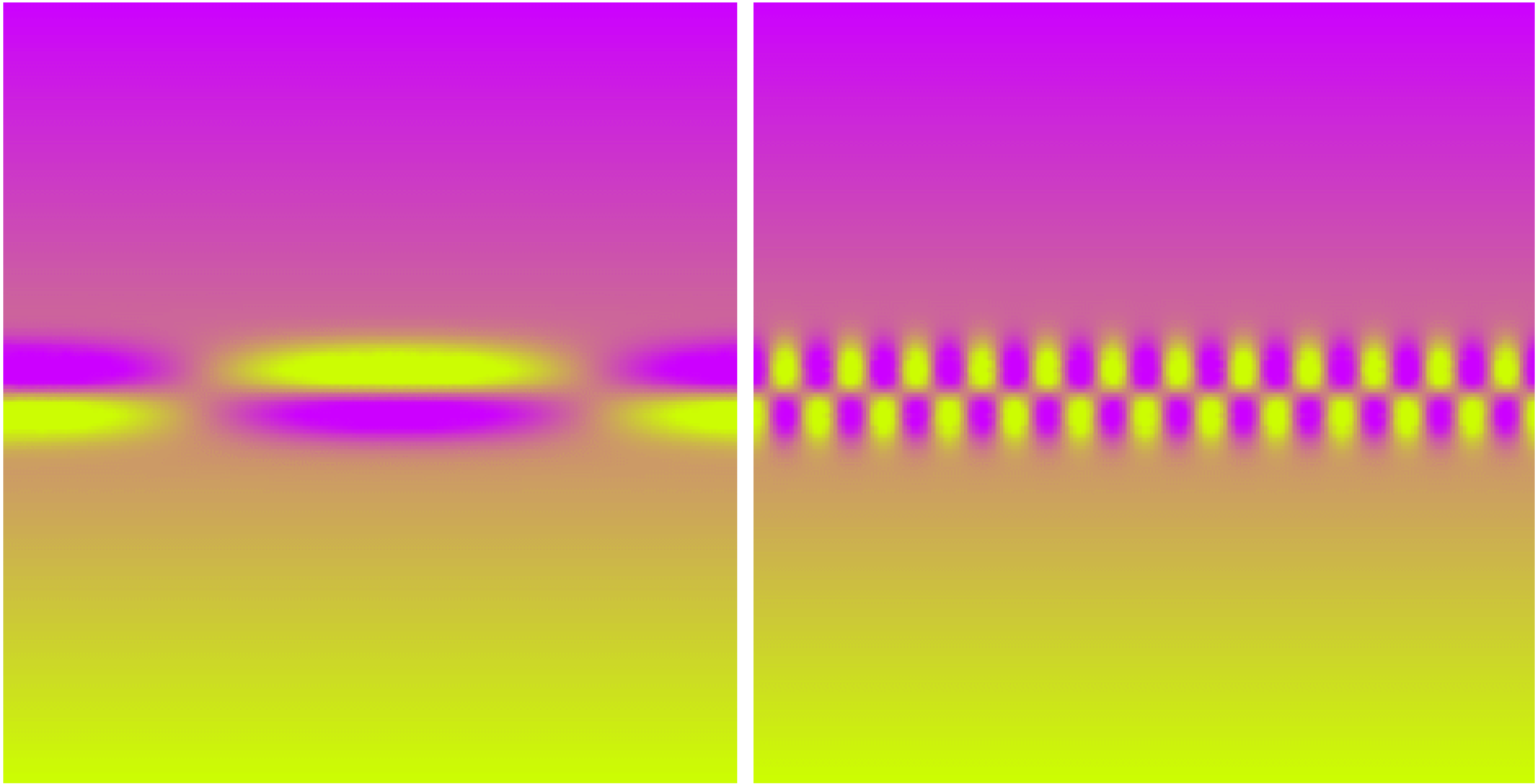
Larger accretion rate

Convection, no diffusion.



Effectiveness is almost independent on scale

Convection, diffusion is on.



Only large scale perturbations survive

Sgr A* observations

$M = 3.6 \cdot 10^{6}\ M_{\Theta}$ (Ghez et.al 2005)

Wind accretion (Cuadra et al, 2006)

Accretion rate can be inferred from

✓Radiation efficiency + IR data (Sharma et al. 2007) from Keck & VLT

✓Direct radio-frequency probing (Bower et al. 2006) with VLBI

✓Faraday rotation measure (Marrone et al. 2007) with SMA

Lots of uncertainties

❑Uncertain distribution of energy between electrons and nuclei
❑Unknown synchrotron emission bunching ratio
❑Undetermined magnetic field structure

All above agree on about $10^{-7} \div 10^{-8}\ M_{\Theta}\,year^{-1} \sim 0.01\,\dot{M}_{B}$

Conclusions

- Accretion rate is much lower, than without magnetic field:
  - 2-5 times without angular momentum
  - Up to 100 times with angular momentum
    - (consistent with Sgr A* observations)
- Many different effects influence the results:
  - Equation of State
  - Finite magnetic helicity
- Convection and diffusion should be accounted for together.
  - Small scale approach does not work.

Flux tubes accretion, realistic diffusion

Future prospects

- Add diffusion and convection (correct dynamics)
- Add angular momentum into equations (2D problem)
- Add cooling (disks)
- Add general relativity (jets)

Solve full axisymmetric problem