ANGULAR MOMENTUM FOR ACCRETION ONTO SGR A*

First of all, I should admit the problem is non-trivial. We with Vasily Beskin still have different answers, but I think It would be better for you to know what we have now.

1. My solution.

Let us consider (x, y, z) Cartesian coordinate system. y is directed to the black hole, x is antiparallel to the speed of the star, z is perpendicular to x and y.

V_w - wind speed in the reference frame of star. v_s - speed of the star. $R = kGM/V^2$ - capture radius ($k \approx 2$). D - orbit radius.

$\sin\alpha = v_s/V_w$ - for the angle of zero angular momentum.

$\delta\phi$ and $\delta\theta$ are polar angles ($\delta\theta$ equals 0 on the equator of the star, $\delta\phi$ equals 0 on the line of zero angular momentum).

Speed of wind: $v_x = V_w \sin(\alpha + \delta\phi)\cos(\delta\theta) - v_s$, $v_y = V_w \cos(\alpha + \delta\phi)\cos(\delta\theta)$, $v_z = V_w \sin\delta\theta$

We make a series below in $\delta\phi$ and $\delta\theta$ for all quantities: $v_x = V_w\delta\phi(1 - \alpha\delta\phi/2)$, $v_y = V_w(1 - \alpha\delta\phi)$, $v_z = V_w\delta\theta$.

Then we try to understand what we want to calculate.

$$l_{eff} = \frac{\int_\phi \int_\theta \dot{m} v_x D d(\delta\phi) d(\delta\theta)}{\int_\phi \int_\theta \dot{m} d(\delta\phi) d(\delta\theta)}. \tag{1}$$

$\dot{m} = const$ is the mass detaching per unit time from unit angle from the surface of the star. Is this expression correct?

If yes, then we continue. $R^* = R\frac{V_w^2}{V_{tot}^2}$ - capture radius of speed, where $V_{tot} = V_w(1 - \alpha\delta\phi)$ - total speed of the wind.

We receive for the boundary $\sin\varepsilon = R^*/D = \sqrt{v_x^2 + v_z^2}/V_{tot}$. Finally:

$$\delta\phi^2(1 - \alpha\delta\phi) + \delta\theta^2 = R^2/D^2(1 + 2\alpha\delta\phi). \tag{2}$$

Then we change variables: $\delta\phi = r\sin\beta$, $\delta\theta = r\cos\beta$.

Making series for r like $r = r_0(1+\delta)$, where $r_0 = R/D$ we find $r_x = r_0(1+\alpha r_0(\sin\beta + \sin^3\beta/2))$.

We can write the following instead of the numerator of (1):

$$\dot{m} V_w D \int_0^{2\pi} \sin\beta d\beta \int_0^{r_x} r\left(1 - \frac{\alpha r \sin\beta}{2}\right) r dr = \frac{5}{4}\pi v_s D (R/D)^4 \dot{m},$$

and $\pi(R/D)^2\dot{m}$ instead of denominator. So,

$$l_{eff} = \frac{5}{4} v_s D \left(\frac{R}{D}\right)^2. \tag{answer}$$

The sign of the effect in negative, i.e. black receive the angular momentum antiparallel to the angular momentum of the star.

Shcherbakov Roman, February 7, 2006.

2. Not my solution.

Vasily Semyonovich writes

$$l_{eff} = \frac{\int_\phi \int_\theta (v_n/V_{tot}) v_x D d(\delta\phi) d(\delta\theta)\dot{m}}{\int_\phi \int_\theta (v_n/V_w)\dot{m} d(\delta\phi) d(\delta\theta)}, \tag{1'}$$

thus trying to consider the fluxes of the angular momentum. v_n/V_{tot} is the projection of the wind speed to the normal vector to the surface of the star. The final coefficient is 1 instead of my $\frac{5}{4}$.